COST E55 Workshop
Graz University of Technology
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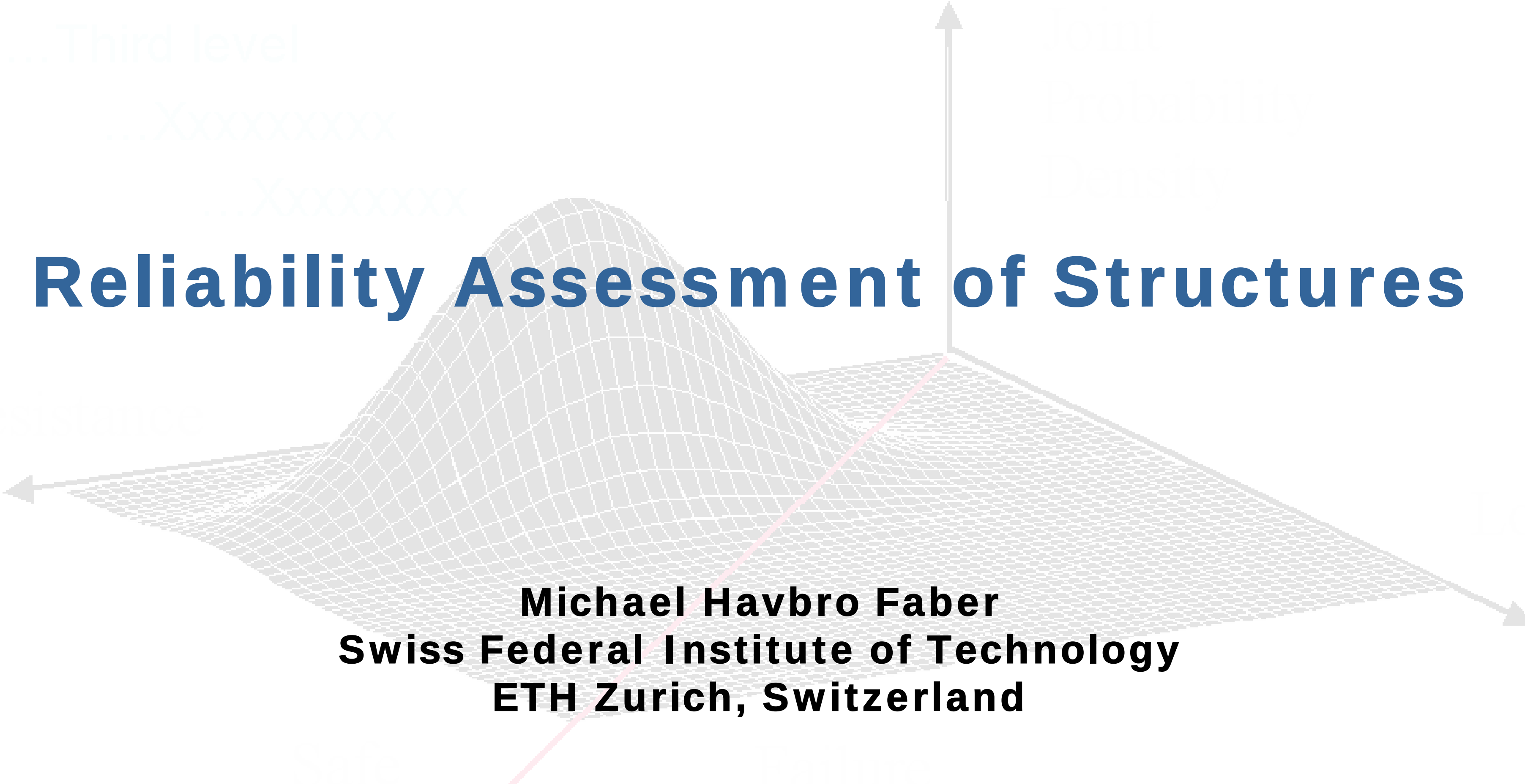

Reliability Assessment of Structures

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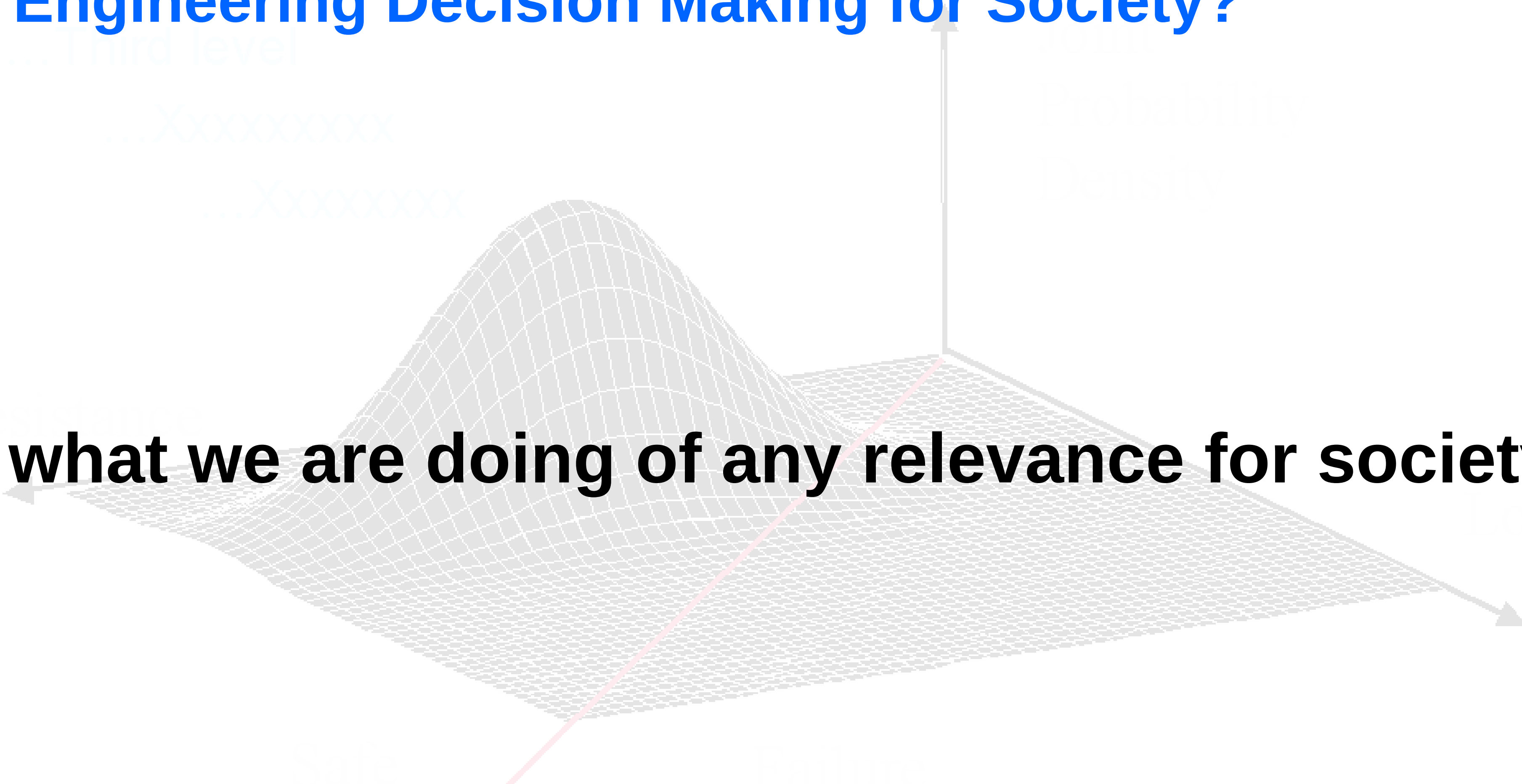
Contents of Presentation

- **An introduction – what is the role of risk and reliability in engineering?**
- **Refreshing you memory on probability and statistics**
- **(the very) Basics of modern reliability theory**
- **Reliability based calibration of design codes**
- **The JCSS approach to risk assessment of engineered facilities**
- **On the issues of risk acceptance – how safe is safe enough?**

Engineering Decision Making for Society?



Is what we are doing of any relevance for society?

Engineering Decision Making for Society?

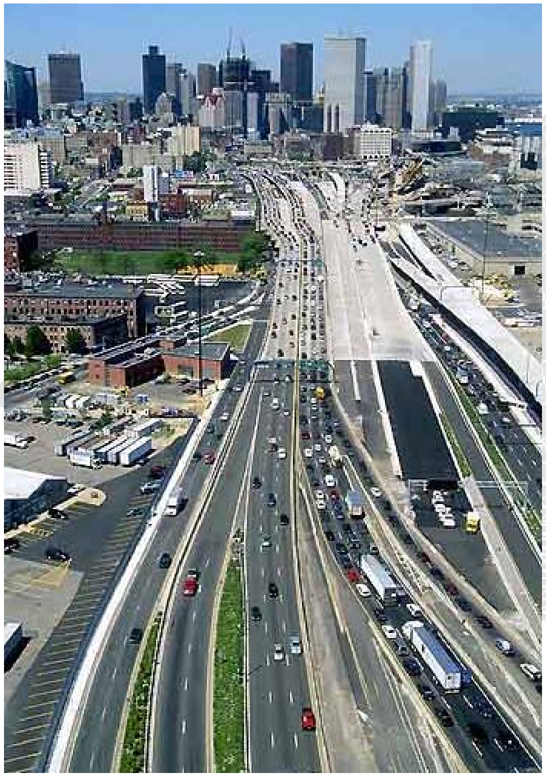
- **Examples of what we help to develop**

Golden Gate Bridge - USA

Øresund bridge - Denmark

Engineering Decision Making for Society?

- **Examples of what we help to develop**



Big Dig Boston/USA

Engineering Decision Making for Society?

- **Examples of what we help to develop**

Hoover Dam - USA

Engineering Decision Making for Society?

- **Examples of what we help to develop**

Hong Kong Island - China

Engineering Decision Making for Society?

- **Helping to control risks due to Natural Hazards**

Tornados and strong winds

Engineering Decision Making for Society?

- **Helping to control risks due to Natural Hazards**

Earthquakes

Engineering Decision Making for Society?

- **Helping to control risks due to degradation**

Corrosion

Fatigue

Engineering Decision Making for Society?

- **Helping to control risks due to accidents**

Fires

Explosions

Engineering Decision Making for Society?

- **Helping to control risks due to malevolence**

Bombs

Airplane impacts

Engineering Decision Making for Society?

- **Helping to reduce consequences of “unfulfilled assumptions”**

Extreme loads/deterioration
Bad Reichenhalle

Design/execution errors
Siemens Arena

Definition of Risk

Risk is a characteristic of an activity relating to all possible events n_E which may follow as a result of the activity

The risk contribution R_{E_i} from the event E_i is defined through the product between

the **Event probability** P_{E_i}

and

the **Consequences** of the event C_{E_i}

The **Risk** associated with a given activity R_A may then be written as

$$R_A = \sum_{i=1}^{n_E} R_{E_i} = \sum_{i=1}^{n_E} P_{E_i} \cdot C_{E_i}$$

Decision Problems in Engineering

Uncertainties must be considered in the decision making throughout all phases of the life of an engineering facility

Uncertainties

Traffic volume

Loads

Resistances (material, soil,..)

Degradation processes

Service life

Manufacturing costs

Execution costs

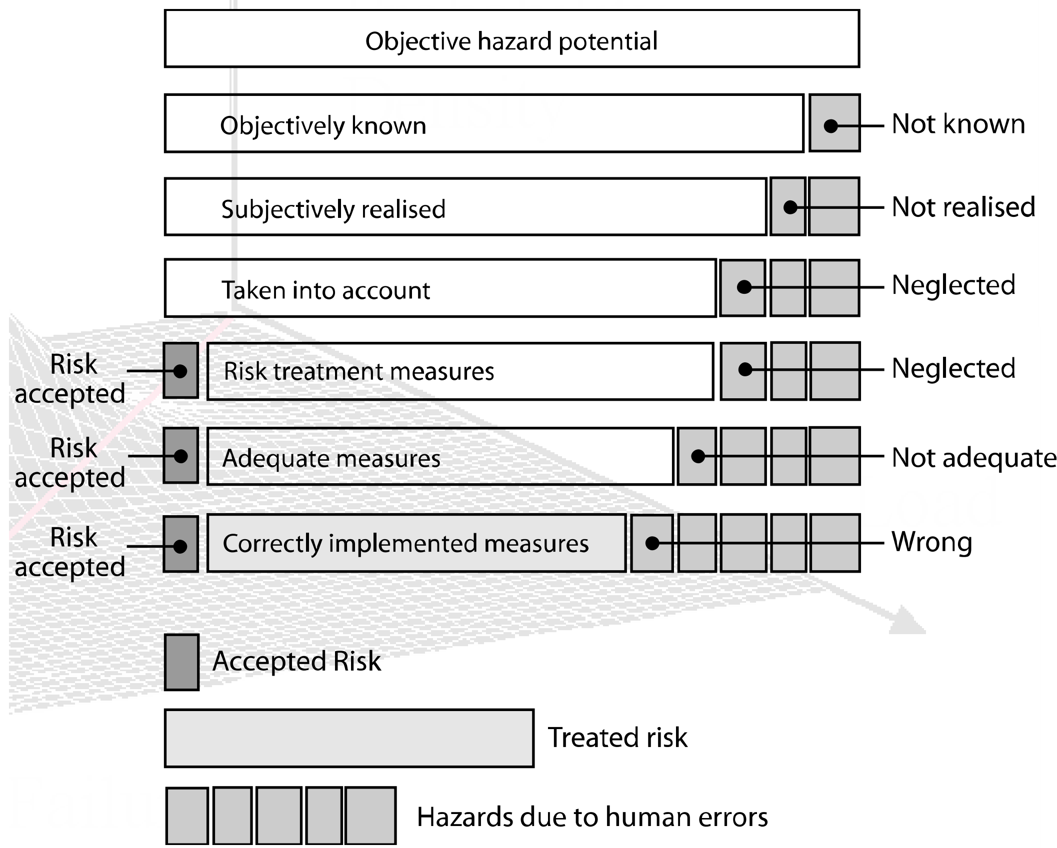
Decommissioning costs

Sources of Risks in Engineering

Any activity carries a risk potential

It is important that this potential is fully understood

Only when the risk potential is fully understood can rational decisions be identified and implemented



Overview of Probability Theory

- What are we aiming for ?

Data ⇄ Model estimation

↓

Probabilistic model

↓

Probabilities of events

Consequences of events

↓

Risks

↓

Decision Making !

The probability theory provides the basis for the consistent treatment of uncertainties in decision making !

We need to be able to quantify the probability of events and to update these based on new information

Interpretation of Probability

States of nature of which we have interest such as:

- a bridge failing due to excessive traffic loads
- a water reservoir being over-filled
- an electricity distribution system „falling out"
- a project being delayed

are in the following denoted „events"

we are generally interested in quantifying the probability that such events take place within a given „time frame"

Interpretation of Probability

- There are in principle three different interpretations of probability
  - Frequentistic $P(A) = \lim \dfrac{N_A}{n_{\exp}}$ **for** $n_{\exp} \rightarrow \infty$
  - Classical $P(A) = \dfrac{n_A}{n_{tot}}$
  - Bayesian $P(A) =$ degree of belief that A will occur

Conditional Probability and Bayes's Rule

as there is $P(A \cap E_i) = P(A|E_i)P(E_i) = P(E_i|A)P(A)$

we have

Likelihood

Prior

$$P(E_i|A) = \frac{P(A|E_i)P(E_i)}{P(A)} = \frac{P(A|E_i)P(E_i)}{\sum_{i=1}^{n} P(A|E_i)P(E_i)}$$

Posterior

Bayes Rule

Reverend Thomas Bayes
(1702-1764)

Uncertainties in Engineering Problems

Different types of uncertainties influence decision making

- Inherent natural variability – aleatory uncertainty
  - result of throwing dices
  - variations in material properties
  - variations of wind loads
  - variations in rain fall

- Model uncertainty – epistemic uncertainty
  - lack of knowledge (future developments)
  - inadequate/imprecise models (simplistic physical modelling)

- Statistical uncertainties – epistemic uncertainty
  - sparse information/small number of data

Uncertainties in Engineering Problems

- Consider as an example a dike structure

  - the design (height) of the dike will be determining the frequency of floods

  - if exact models are available for the prediction of future water levels and our knowledge about the input parameters is perfect then we can calculate the frequency of floods (per year) - a deterministic world !

  - even if the world would be deterministic – we would not have perfect information about it – so we might as well consider the world as random

Uncertainties in Engineering Problems

In principle the so-called

inherent physical uncertainty (aleatory - Type I)

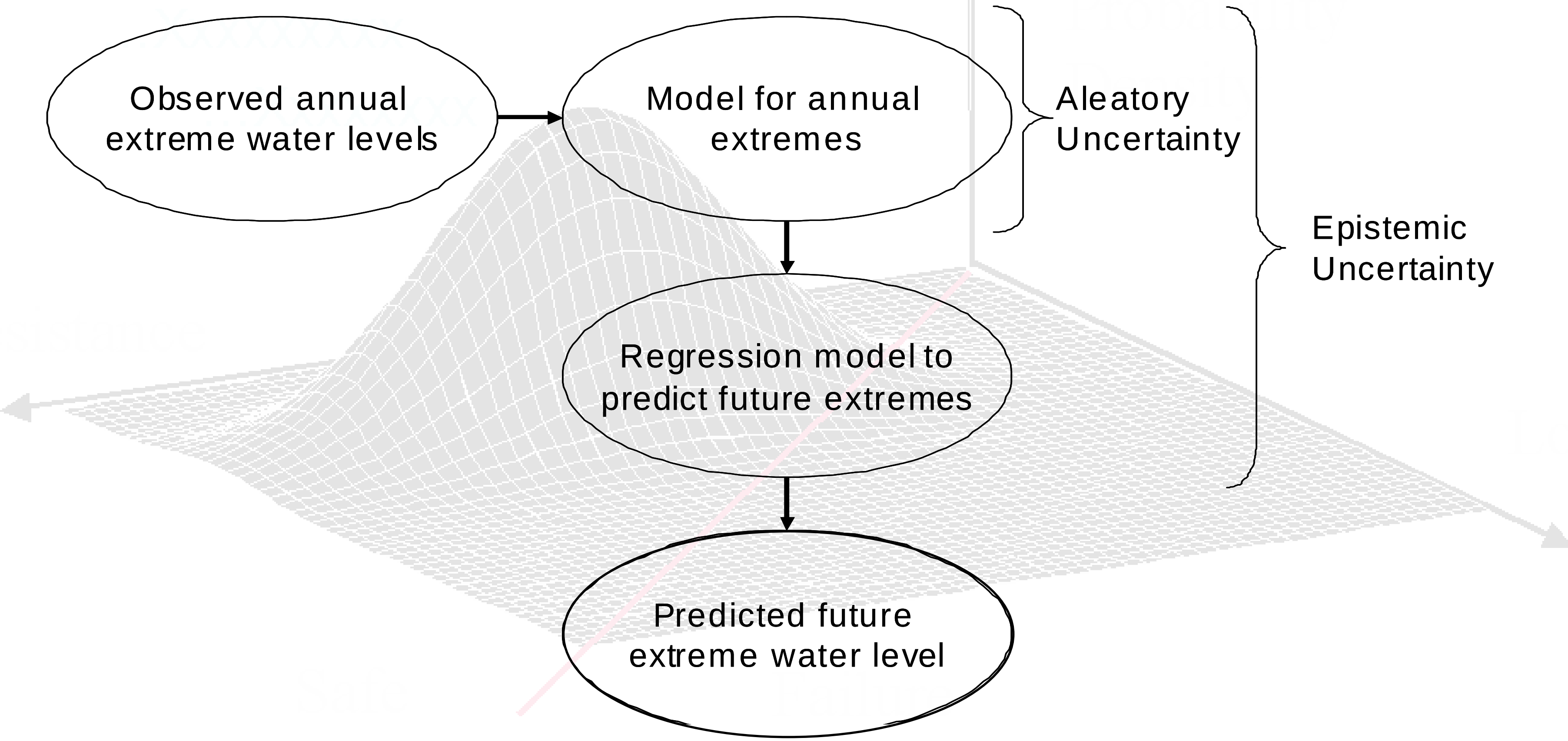
is the uncertainty caused by the fact that the world is random, however, another pragmatic viewpoint is to define this type of uncertainty as

any uncertainty which cannot be reduced by means of collection of additional information

the uncertainty which can be reduced is then the

model and statistical uncertainties (epistemic - Type II)

Uncertainties in Engineering Problems



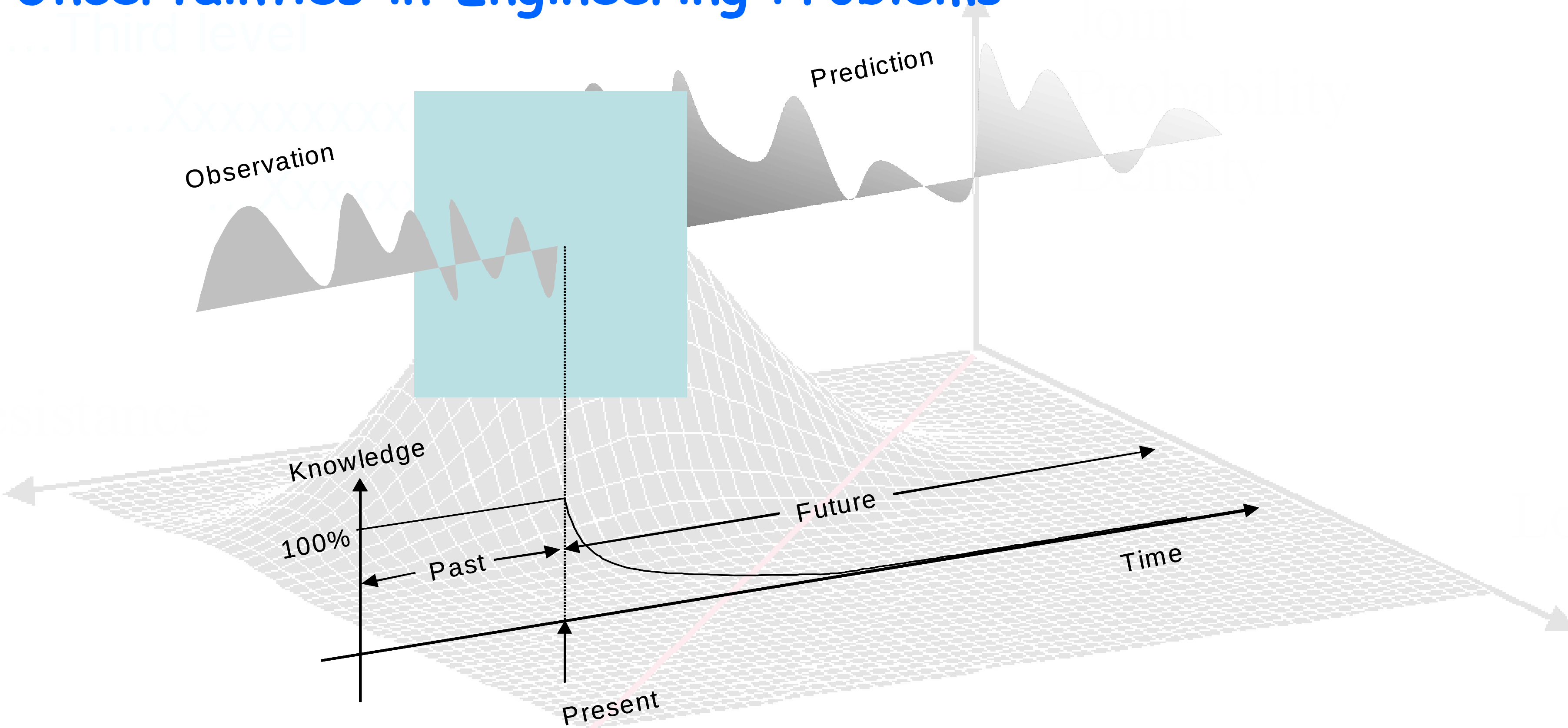
Uncertainties in Engineering Problems

The relative contribution of aleatory and epistemic uncertainty to the prediction of future water levels is thus influenced directly by the applied models

refining a model might reduce the epistemic uncertainty – but in general also changes the contribution of aleatory uncertainty

the uncertainty structure of a problem can thus be said to be scale dependent !

Uncertainties in Engineering Problems



The uncertainty structure changes also as function of time – is thus time dependent !

Random Variables

- Probability distribution and density functions

A random variable is denoted with capital letters : X

A realization of a random variable is denoted with small letters : x

We distinguish between

- *continuous random variables* : can take any value in a given range
- *discrete random variables* : can take only discrete values

Random Variables

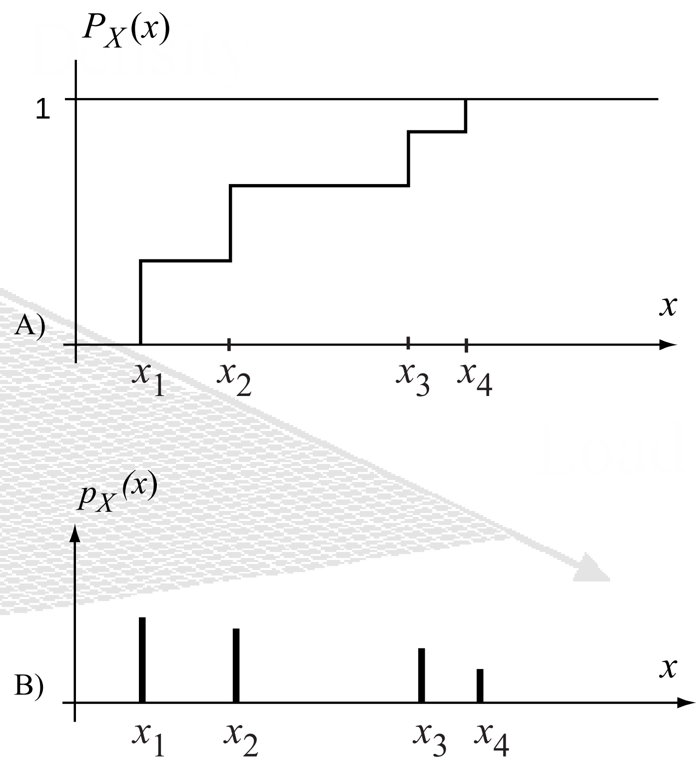
- **Probability distribution and density functions**

  The probability that the outcome of a discrete random variable X is smaller than x is denoted the *probability distribution function*

$$P_X(x) = \sum_{x_i < x} p_X(x_i)$$

  The *probability density function* for a discrete random variable is defined by

$$p_X(x_i) = P(X = x)$$



Random Variables

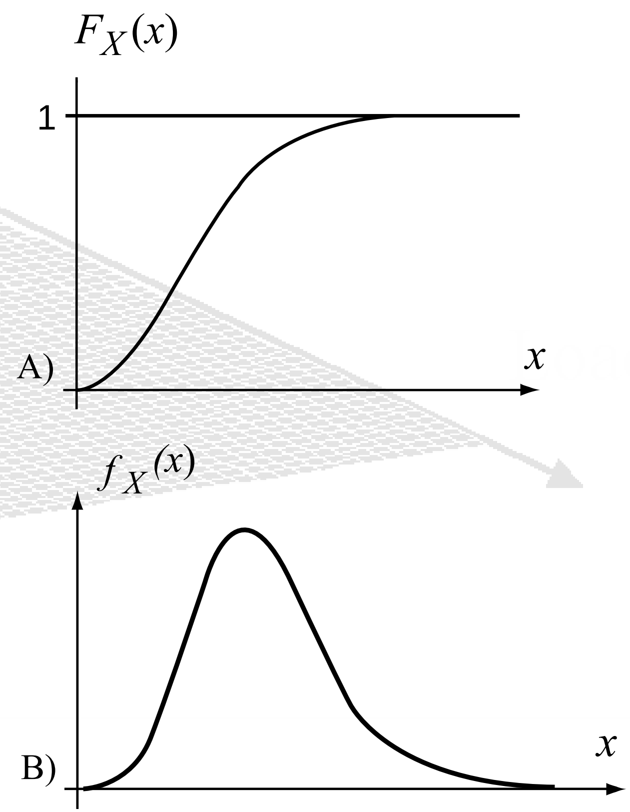
- **Probability distribution and density functions**

  The probability that the outcome of a continuous random variable X is smaller than x is denoted the ***probability distribution function***

$$F_X(x) = P(X < x)$$

The probability density function for a continuous random variable is defined by

$$f_X(x) = \frac{\partial F_X(x)}{\partial x}$$



Random Variables

- Moments of random variables and the expectation operator

  Probability distribution and density function can be described in terms of their parameters $\mathbf{p}$ or their moments

  Often we write

$$F_X(x,\mathbf{p}) \qquad f_X(x,\mathbf{p})$$

Parameters

The parameters can be related to the moments and visa versa

Random Variables

- Moments of random variables and the expectation operator

  The i'th moment m_i for a continuous random variable X is defined through

$$m_i = \int_{-\infty}^{\infty} x^i \cdot f_X(x)dx$$

  The *expected value E[X]* of a continuous random variable X is defined accordingly as the first moment

$$\mu_X = E[X] = \int_{-\infty}^{\infty} x \cdot f_X(x)dx$$

Random Variables

- Moments of random variables and the expectation operator

The i'th moment m_i for a discrete random variable X is defined through

$$m_i = \sum_{j=1}^{n} x_j^i \cdot p_X(x_j)$$

The *expected value E[X]* of a discrete random variable X is defined accordingly as the first moment

$$\mu_X = E\left[X\right] = \sum_{j=1}^{n} x_j \cdot p_X(x_j)$$

Random Variables

- Moments of random variables and the expectation operator

  The *standard deviation* σ_X of a continuous random variable is defined as the second central moment i.e. for a continuous random variable *X* we have

$$\underbrace{\sigma_X{}^2 = \mathrm{Var}[\mathrm{X}]}_{\textbf{Variance}} = E\left[(X-\mu_X)^2\right] = \int_{-\infty}^{\infty} (x-\underbrace{\mu_X}_{\textbf{Mean value}})^2 \cdot f_X(x)dx$$

for a discrete random variable we have correspondingly

$$\sigma_X^2 = Var[X] = \sum_{j=1}^{n} (x_j - \mu_X)^2 \cdot p_X(x_j)$$

Random Variables

- **Moments of random variables and the expectation operator**

The ratio between the standard deviation and the expected value of a random variable is called the *Coefficient of Variation CoV* and is defined as

$$CoV\left[X\right]=\frac{\sigma_X}{\mu_X}$$

Dimensionless

a useful characteristic to indicate the variability of the random variable around its expected value

Random Variables

- Typical probability distribution functions in engineering

  Normal : sum of random effects

  Log-Normal: product of random effects

  Exponential: waiting times

  Gamma: Sum of waiting times

  Beta: Flexible modeling function

Distribution type	Parameters	Moments
Rectangular $a \le x \le b$ $f_X(x)=\frac{1}{b-a}$ $F_X(x)=\frac{x-a}{b-a}$	a b	$\mu=\frac{a+b}{2}$ $\sigma=\frac{b-a}{\sqrt{12}}$
Normal $f_X(x)=\frac{1}{\sigma\sqrt{2\pi}}exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right)$ $F_X(x)=\frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{x}exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right)dx$	μ $\sigma>0$	μ σ
Shifted Lognormal $x>\varepsilon$ $f_X(x)=\frac{1}{(x-\varepsilon)\zeta\sqrt{2\pi}}\exp\left(-\frac{1}{2}\left(\frac{\ln(x-\varepsilon)-\lambda}{\zeta}\right)^2\right)$ $F_X(x)=\Phi\left(\frac{ln(x-\varepsilon)-\lambda}{\zeta}\right)^2$	λ $\zeta>0$ ε	$\mu=\varepsilon+\exp\left(\lambda+\frac{\zeta^2}{2}\right)$ $\sigma=\exp\left(\lambda+\frac{\zeta^2}{2}\right)\sqrt{\exp(\zeta^2)-1}$
Shifted Exponential $x\ge\varepsilon$ $f_X(x)=\lambda\exp(-\lambda(x-\varepsilon))$ $F_X(x)=1-e^{-\lambda(x-\varepsilon)}$	ε $\lambda>0$	$\mu=\varepsilon+\frac{1}{\lambda}$ $\sigma=\frac{1}{\lambda}$
Gamma $x\ge 0$ $f_X(x)=\frac{b^p}{\Gamma(p)}\exp(-bx)x^{p-1}$ $F_X(x)=\frac{\Gamma(bx,p)}{\Gamma(p)}$	$p>0$ $b>0$	$\mu=\frac{p}{b}$ $\sigma=\frac{\sqrt{p}}{b}$
Beta $a\le x\le b,\ r,t\ge 1$ $f_X(x)=\frac{\Gamma(r+t)}{\Gamma(r)\cdot\Gamma(t)}\frac{(x-a)^{r-1}(b-x)^{t-1}}{(b-a)^{r+t-1}}$ $F_X(x)=\frac{\Gamma(r+t)}{\Gamma(r)\cdot\Gamma(t)}\cdot\int_a^u\frac{(u-a)^{r-1}(b-u)^{t-1}}{(b-a)^{r+t-1}}du$	a b $r>1$ $t>1$	$\mu=a+(b-a)\frac{r}{r+1}$ $\sigma=\frac{b-a}{r+t}\sqrt{\frac{rt}{r+t+1}}$

Stochastic Processes and Extremes

- Random quantities may be “time variant” in the sense that they take new values at different times or at new trials.

  - If the new realizations occur at discrete times and have discrete values the random quantity is called a random sequence

    failure events, traffic congestions,…

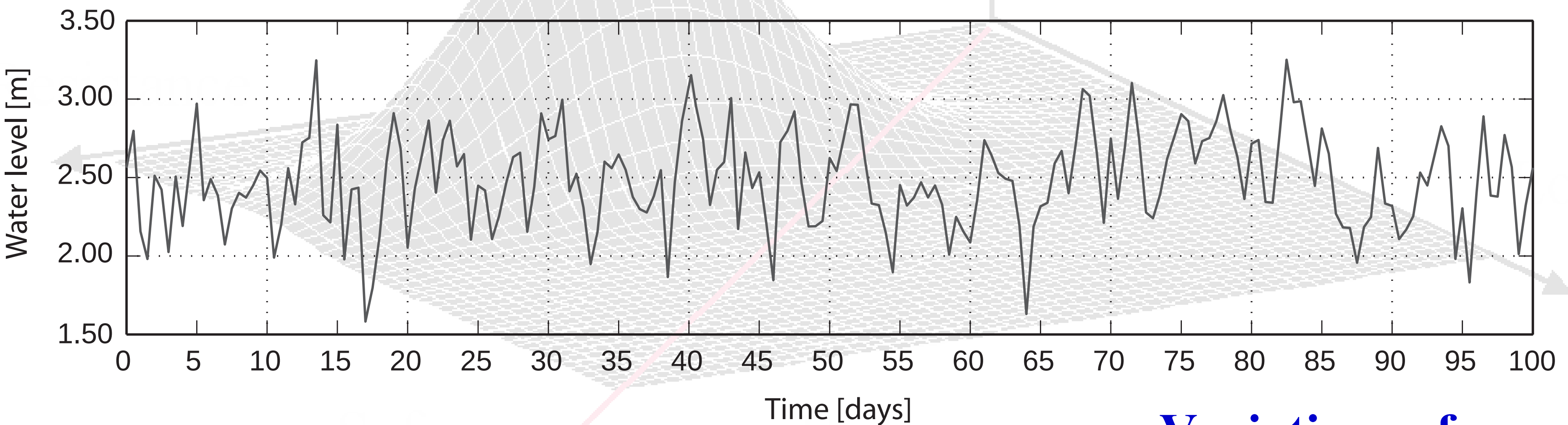
  - If the new realizations occur continuously in time and take continuous values the random quantity is called a random process or stochastic process

    wind velocity, wave heights,…

Stochastic Processes and Extremes

- **Continuous random processes**

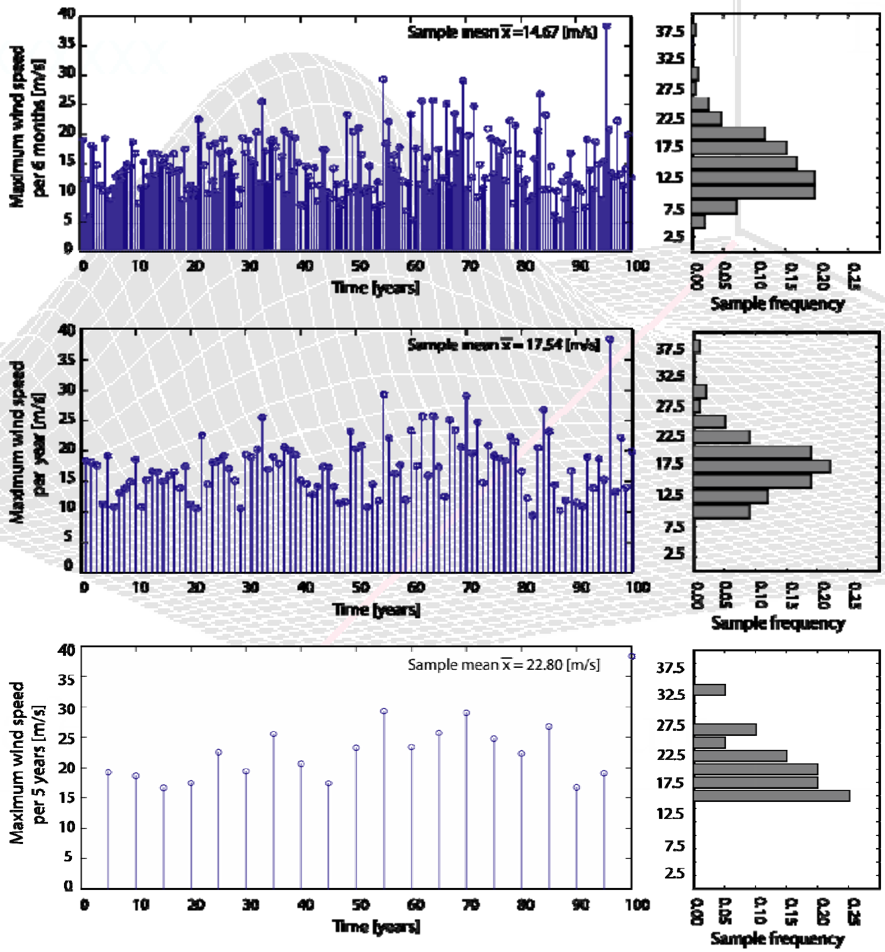
  A continuous random process is a random process which has realizations continuously over time and for which the realizations belong to a continuous sample space.



Variations of:
water levels
wind speed
rain fall

Stochastic Processes and Extremes

Extremes of a random process:



Stochastic Processes and Extremes

Extreme Value Distributions

In engineering we are often interested in extreme values i.e. the smallest or the largest value of a certain quantity within a **certain time interval** e.g.:

The largest earthquake in 1 year

The highest wave in a winter season

The largest rainfall in 100 years

Stochastic Processes and Extremes

Extreme Value Distributions

We could also be interested in the smallest or the largest value of a certain quantity within a **certain volume or area** unit e.g.:

- The largest concentration of pesticides in a volume of soil
- The weakest link in a chain
- The smallest thickness of concrete cover

Extreme Value Distributions

If the extremes within the period *T* of an ergodic random process *X*(*t*) are independent and follow the distribution:

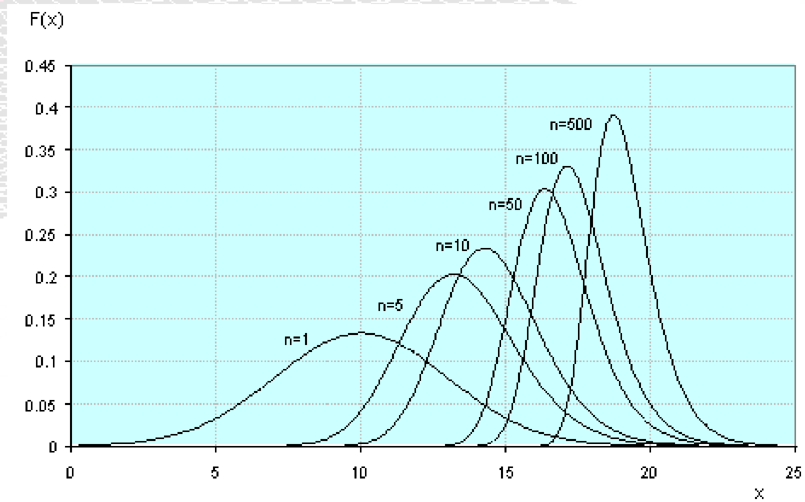
$$F_{X,T}^{\max}(x)$$

Then the extremes of the same process within the period:

$$n \cdot T$$

will follow the distribution:

$$F_{X,nT}^{\max}(x) = \left(F_{X,T}^{\max}(x)\right)^n$$



Extreme Value Distributions

Extreme Type I – Gumbel Max

When the upper tail of the probability density function falls off exponentially (exponential, Normal and Gamma distribution) then the maximum in the time interval *T* is said to be Type I extreme distributed

$$f_{X,T}^{\max}(x) = \alpha \exp(-\alpha(x-u) - \exp(-\alpha(x-u)))$$

$$F_{X,T}^{\max}(x) = \exp(-\exp(-\alpha(x-u)))$$

$$\mu_{X_T^{\max}} = u + \frac{\gamma}{\alpha} = u + \frac{0.577216}{\alpha}$$

$$\sigma_{X_T^{\max}} = \frac{\pi}{\alpha\sqrt{6}}$$

For increasing time intervals the variance is constant but the mean value increases as:

$$\mu_{X_{nT}^{\max}} = \mu_{X_T^{\max}} + \frac{\sqrt{6}}{\pi}\sigma_{X_T^{\max}} \ln(n)$$

Extreme Value Distributions

Extreme Type II – Frechet Max

When a probability density function is downwards limited at zero and upwards falls off in the form

$$F_X(x) = 1 - \beta\left(\frac{1}{x}\right)^k$$

then the maximum in the time interval *T* is said to be Type II extreme distributed

$$F_{X,T}^{\max}(x) = \exp\left(-\left(\frac{u}{x}\right)^k\right)$$

$$f_{X,T}^{\max}(x) = \frac{k}{u}\left(\frac{u}{x}\right)^{k+1} \exp\left(-\left(\frac{u}{x}\right)^k\right)$$

Mean value and standard deviation

$$\mu_{X_T^{\max}} = u\Gamma\left(1 - \frac{1}{k}\right)$$

$$\sigma_{X_T^{\max}}^2 = u^2\left[\Gamma\left(1 - \frac{2}{k}\right) - \Gamma^2\left(1 - \frac{1}{k}\right)\right]$$

Extreme Value Distributions

Extreme Type III – Weibull Min

When a probability density function is downwards limited at ε and the lower tail falls off towards ε in the form

$$F(x) = c(x-\varepsilon)^k$$

then the minimum in the time interval *T* is said to be Type III extreme distributed

$$F_{X,T}^{\min}(x) = 1 - \exp\left(-\left(\frac{x-\varepsilon}{u-\varepsilon}\right)^k\right)$$

$$f_{X,T}^{\min}(x) = \frac{k}{u-\varepsilon}\left(\frac{x-\varepsilon}{u-\varepsilon}\right)^{k-1}\exp\left(-\left(\frac{x-\varepsilon}{u-\varepsilon}\right)^k\right)$$

Mean value and standard deviation

$$\mu_{X_T^{\min}} = \varepsilon + (u-\varepsilon)\Gamma(1+\frac{1}{k})$$

$$\sigma_{X_T^{\min}}^2 = (u-\varepsilon)^2\left[\Gamma(1+\frac{2}{k}) - \Gamma^2(1+\frac{1}{k})\right]$$

Stochastic Processes and Extremes

Return period for extreme events:

The return period for extreme events T_R may be defined as

$$T_R = n \cdot T = \frac{1}{(1 - F_{X,T}^{\max}(x))}$$

If the probability of exceeding x during a reference period of 1 year is 0.01 then the return period for exceedances is

$$T_R = n \cdot T = \frac{1}{0.01} = 100 \cdot 1 = 100$$

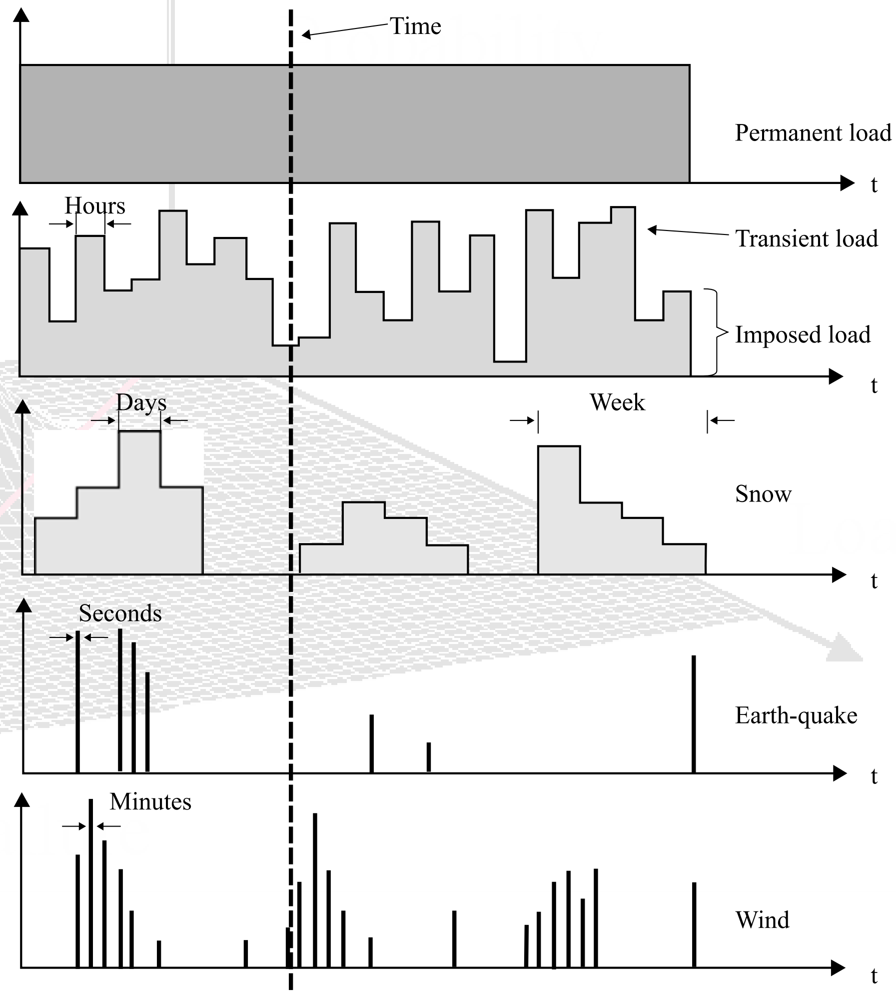
Stochastic Processes and Extremes

- **Loads on Structures**

Combination of loads

We are interested in the maximum of a sum of load effects from different loads

$$X_{max}(T) = \max_{T}\left\{X_1(t) + X_2(t) + \ldots + X_n(t)\right\}$$



Probabilistic Modeling of Resistances

- **The JCSS PMC**

  Part I : Basis of design
  Part II: Load models
  Part III: Resistance models
  Part IV: Examples

Probabilistic Modeling of Resistances

- **The JCSS PMC – Load Models**

2.00	GENERAL PRINCIPLES
2.01	SELF WEIGHT
2.02	LIVE LOAD
2.06	LOADS IN CAR PARKS
2.12	SNOW LOAD
2.13	WIND LOAD
2.15	WAVE LOAD
2.17	EARTHQUAKE
2.18	IMPACT LOAD
2.20	FIRE

Probabilistic Modeling of Resistances

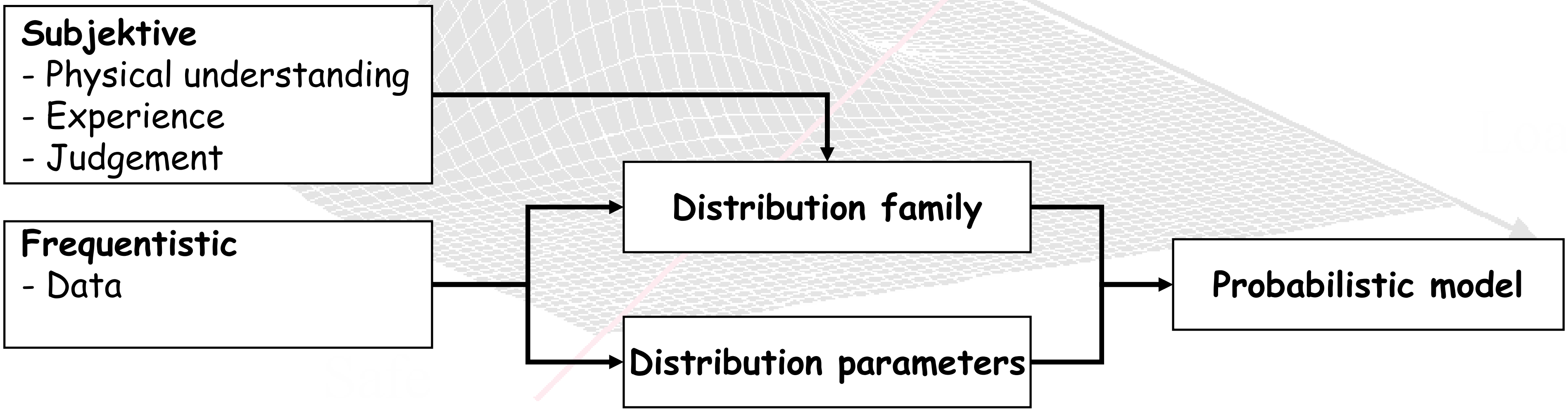
- **The JCSS PMC – Resistance models**

3.00	GENERAL PRINCIPLES
3.01	CONCRETE
3.02	STRUCTURAL STEEL
3.0*	REINFORCING STEEL
3.04	PRESTRESSING STEEL
3.05	**TIMBER**
3.07	SOIL PROPERTIES
3.09	MODELUNCERTAINTIES
3.10	DIMENSIONS
3.11	EXCENTRICITIES

Overview of Estimation and Model Building

Different types of information is used when developing engineering models

- subjektive information
- frequentistic information



Overview of Estimation and Model Building

Model building may be seen to consist of five steps

1) Assessment and statistical quantification of the available data

2) Selection of distribution function

3) Estimation of distribution parameters

4) Model verification

5) Model updating

Structural Reliability Analysis

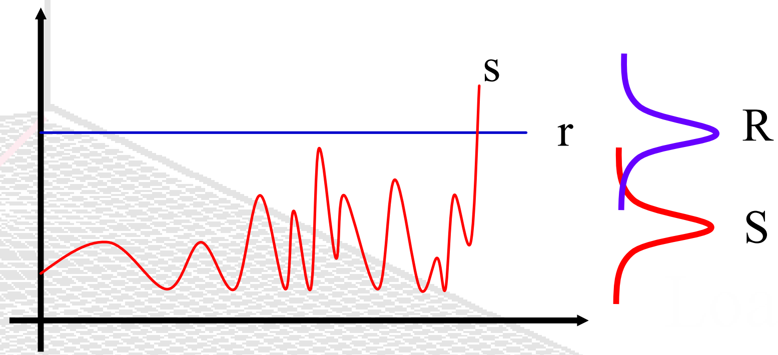
Reliability of structures cannot be assessed through failure rates because

- Structures are unique in nature

- Structural failures normally take place due to extreme loads exceeding the residual strength

Therefore in structural reliability, models are established for resistances R and loads S individually and the structural reliability is assessed through:

$$P_f = P(R - S \leq 0)$$



Structural Reliability Analysis

If only the resistance is uncertain the failure probability may be assessed by

$$P_f = P(R \le s) = F_R(s) = P(R/s \le 1)$$

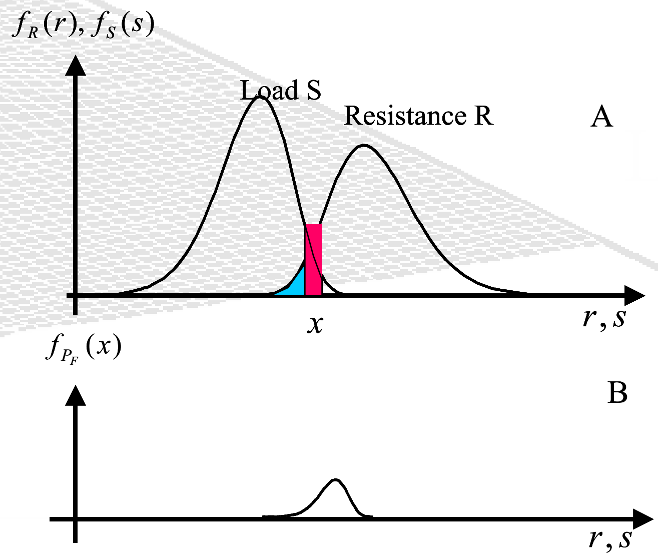
If also the load is uncertain we have

$$P_f = P(R \le S) = P(R - S \le 1) = \int_{-\infty}^{\infty} F_R(x) f_S(x) dx$$

where it is assumed that the load and the resistance are independent

This is called the

„Fundamental Case“



Structural Reliability Analysis

In the case where *R* and *S* are normal distributed the safety margin *M* is also normal distributed

$$M = R - S$$

Then the failure probability is

$$P_F = P(R - S \le 0) = P(M \le 0)$$

with the mean value of M

$$\mu_M = \mu_R - \mu_S$$

and standard deviation of M

$$\sigma_M = \sqrt{\sigma_R^2 + \sigma_S^2}$$

The failure probability is then

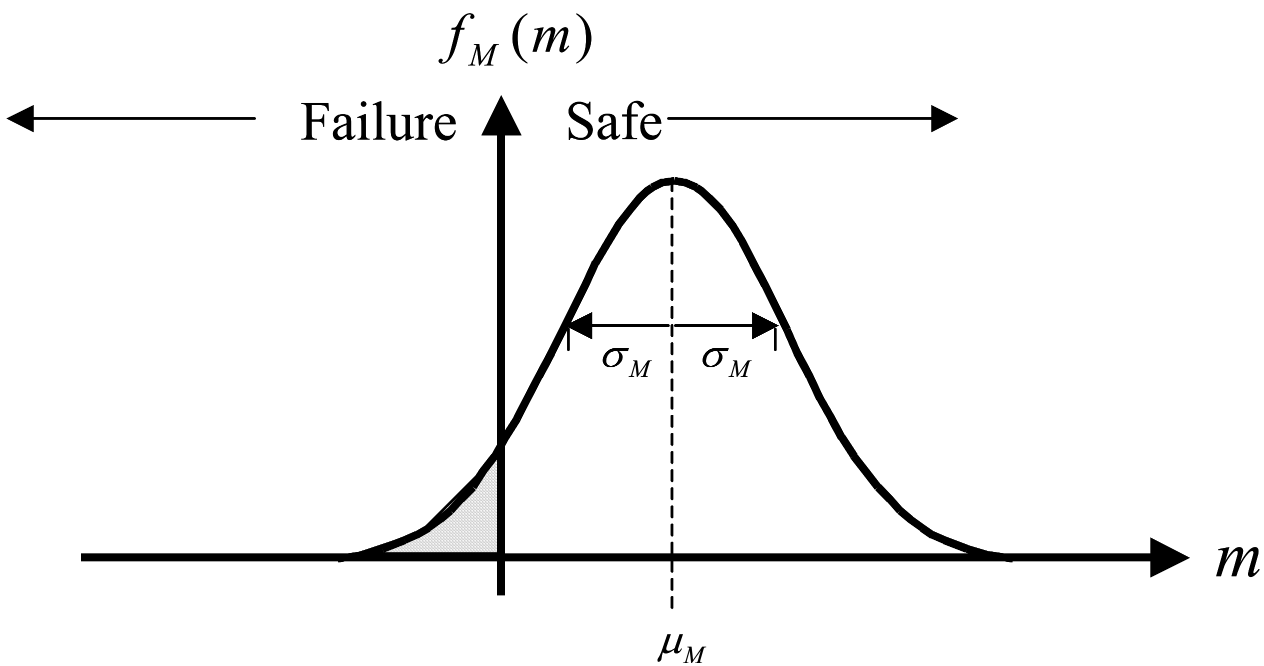
$$P_F = \Phi\left(\frac{0 - \mu_M}{\sigma_M}\right) = \Phi(-\beta)$$

where the reliability index is

$$\beta = \mu_M / \sigma_M$$

Structural Reliability Analysis

The normal distributed safety margin *M*



Structural Reliability Analysis

In the general case the resistance and the load may be defined in terms of functions

$$R = f_1(\mathbf{X})$$

$$S = f_2(\mathbf{X})$$

where $\mathbf{X}$ are basic random variables

and the safety margin as

$$M = R - S = f_1(\mathbf{X}) - f_2(\mathbf{X}) = g(\mathbf{X})$$

where $g(\mathbf{x}) \leq 0$ is called the

limit state function

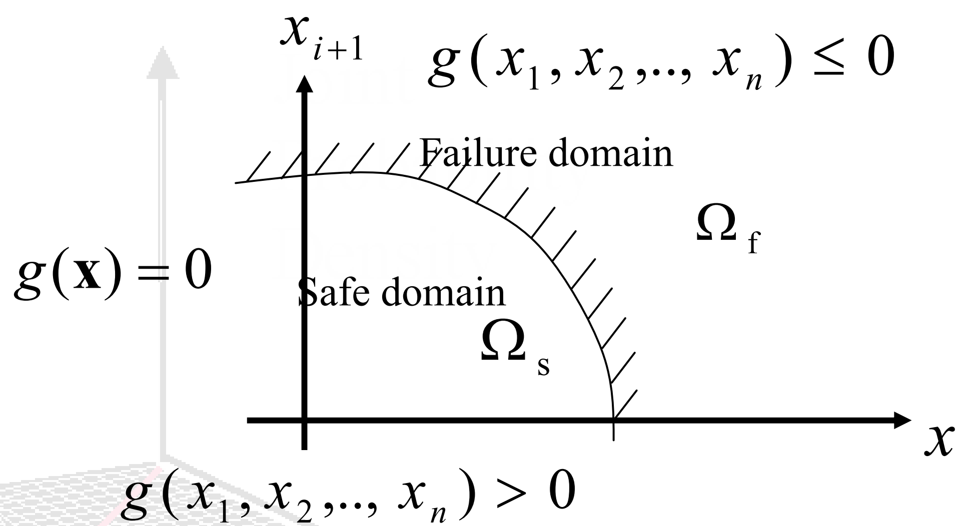
failure occurs when

$$g(\mathbf{x}) \leq 0$$

Structural Reliability Analysis

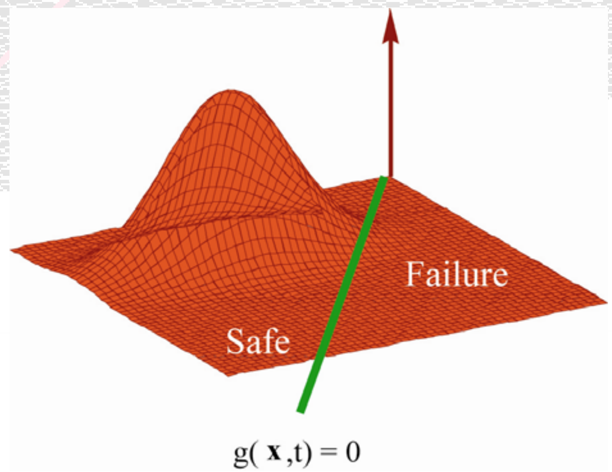
Setting $g(\mathbf{x}) = 0$ defines a (n-1) dimensional surface in the space spanned by the *n* basic variables X

This is the failure surface separating the sample space of X into a safe domain and a failure domain



The failure probability may in general terms be written as

$$P_f = \int_{g(\mathbf{x}) \leq 0} f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x}$$



Failure event

$$\mathbf{F} = \{g(\mathbf{x}) \leq 0\}$$

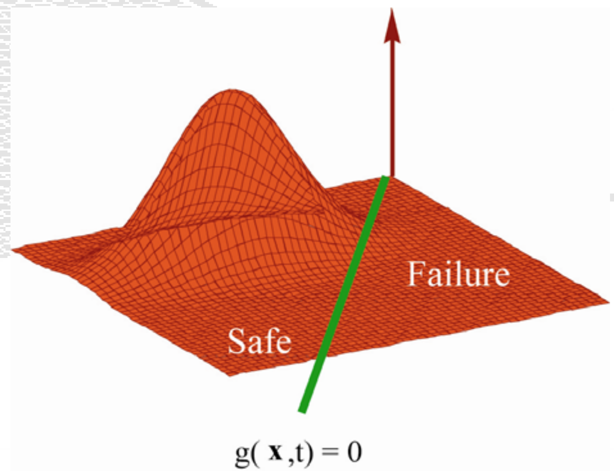
Basics of Structural Reliability Methods

The probability of failure can be assessed by

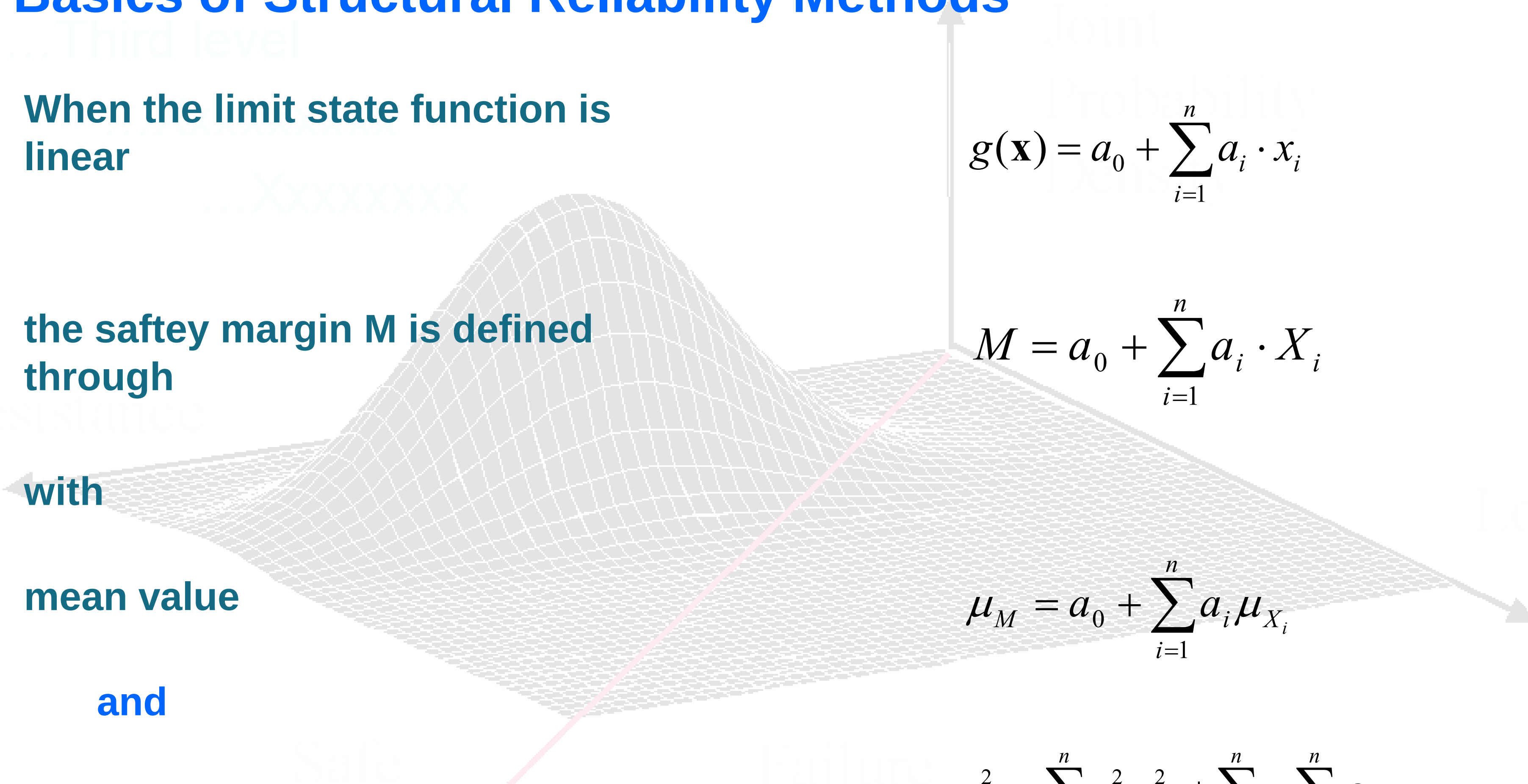
$$P_f = \int_{\Omega_f = \{g(\mathbf{x}) \le 0\}} f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x}$$

where $f_{\mathbf{X}}(\mathbf{x})$ is the joint probability density function for the basic random variables X

For the 2-dimensional case the failure probability simply corresponds to the integral under the joint probability density function in the area of failure



Basics of Structural Reliability Methods



When the limit state function is linear

$$g(\mathbf{x}) = a_0 + \sum_{i=1}^{n} a_i \cdot x_i$$

the saftey margin M is defined through

$$M = a_0 + \sum_{i=1}^{n} a_i \cdot X_i$$

with

mean value

$$\mu_M = a_0 + \sum_{i=1}^{n} a_i \mu_{X_i}$$

and

variance

$$\sigma_M^2 = \sum_{i=1}^{n} a_i^2 \sigma_{X_i}^2 + \sum_{i=1}^{n} \sum_{j=1, j \neq i}^{n} \rho_{ij} a_i a_j \sigma_i \sigma_j$$

Basics of Structural Reliability Methods

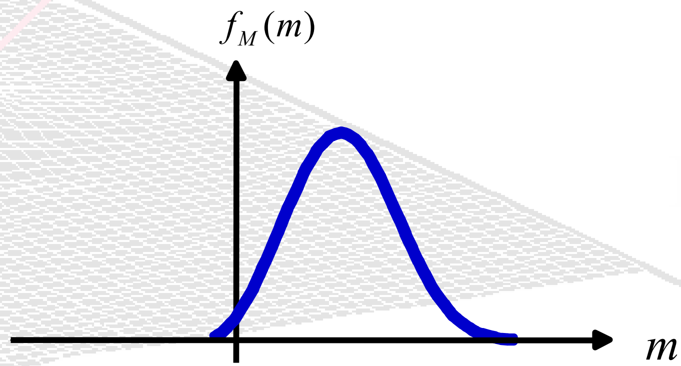
The failure probability can then be written as

$$P_F = P(g(\mathbf{X}) \leq 0) = P(M \leq 0)$$

The reliability index is defined as

$$\beta = \frac{\mu_M}{\sigma_M}$$

Basler and Cornell



Provided that the safety margin is normal distributed

the failure probability is determined as

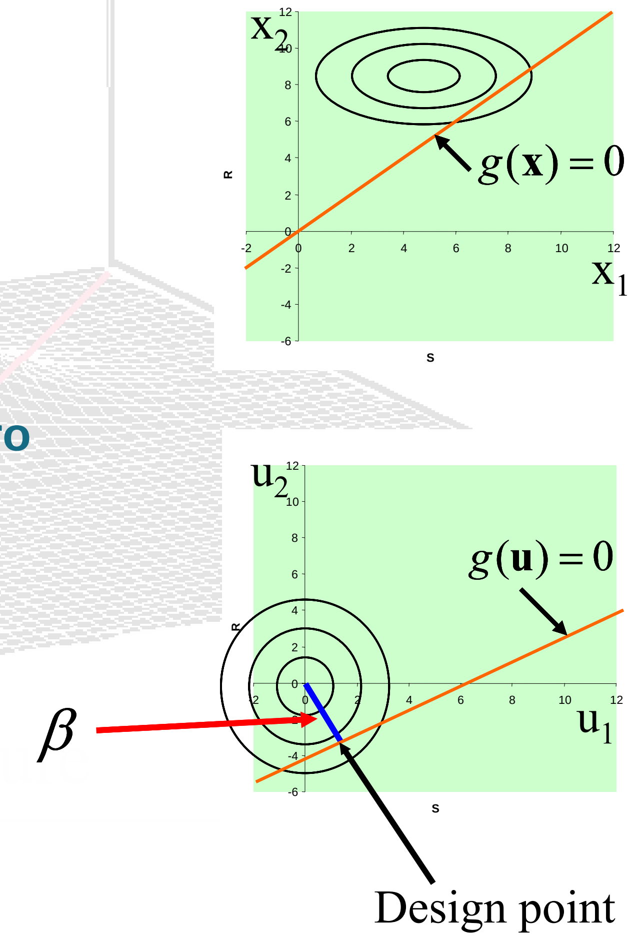
$$P_F = \Phi(-\beta)$$

Basics of Structural Reliability Methods

The reliability index β has the geometrical interpretation of being the shortest distance between the failure surface and the origin in standard normal distributed space u

$$U_i = \frac{X_i - \mu_{X_i}}{\sigma_{X_i}}$$

in which case the components of U have zero means and variances equal to 1



Basics of Structural Reliability Methods

Example:

Consider a steel rod with resistance *r* subjected to a tension force *s*

_r_ and _s_ are modeled by the random variables _R_ and _S_

The probability of failure is wanted

$$g(\mathbf{X}) = R - S$$

$$\mu_R = 350, \sigma_R = 35$$
$$\mu_S = 200, \sigma_S = 40$$

$$P(R - S \leq 0)$$

Basics of Structural Reliability Methods

Example:

Consider a steel rod with resistance *r* subjected to a tension force *s*

$$g(\mathbf{X}) = R - S$$

r* and *s* are modeled by the random variables *R* and *S

$$\mu_R = 350, \sigma_R = 35$$
$$\mu_S = 200, \sigma_S = 40$$

The probability of failure is wanted

$$P(R - S \leq 0)$$

The safety margin is given as

$$M = R - S \begin{cases} \mu_M = 350 - 200 = 150 \\ \sigma_M = \sqrt{35^2 + 40^2} = 53.15 \end{cases}$$

The reliability index is then

$$\beta = \frac{150}{53.15} = 2.84$$

and the probability of failure

$$P_F = \Phi(-2.84) = 2.4 \cdot 10^{-3}$$

Basics of Structural Reliability Methods

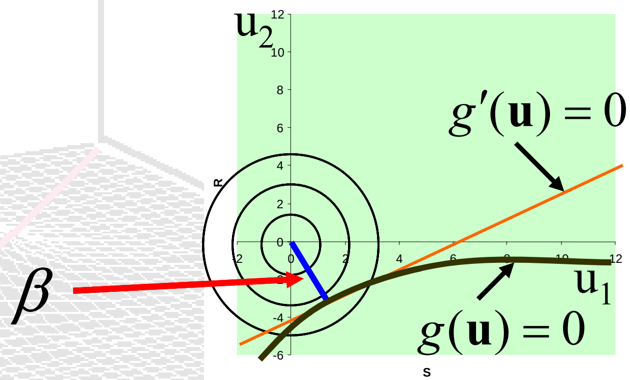
Usually the limit state function is non-linear

- this small phenomenon caused the so-called invariance problem

Hasofer & Lind suggested to linearize the limit state function in the design point

- this solved the invariance problem

Can however easily be linearized !



The reliability index may then be determined by the following optimization problem

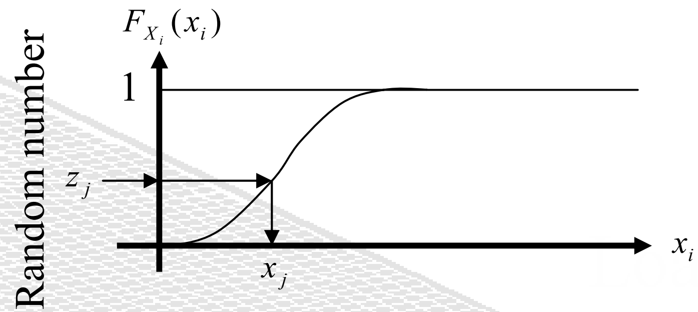
$$\beta = \min_{\mathbf{u} \in \{g(\mathbf{u})=0\}} \sqrt{\sum_{i=1}^{n} u_i^2}$$

Basics of Structural Reliability Methods

Simulation methods may also be used to solve the integration problem

$$P_f = \int_{\Omega_f = \{g(\mathbf{x}) \leq 0\}} f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x}$$

1) *m* realizations of the vector X are generated



2) for each realization the value of the limit state function is evaluated

3) the realizations where the limit state function is zero or negative are counted

$$n_f$$

4) The failure probability is estimated as

$$p_f = \frac{n_f}{m}$$

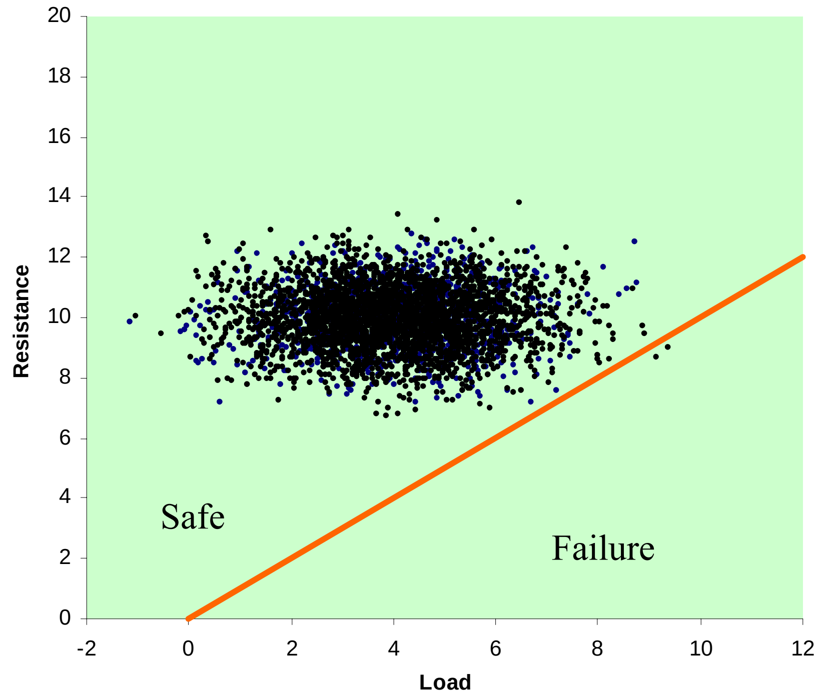
Basics of Structural Reliability Methods

- **Estimation of failure probabilities using Monte Carlo Simulation**

  - ***m*** **random outcomes of R und S are generated and the number of outcomes** n_f **in the failure domain are recorded and summed**

  - **The failure probability** p_f **is then**

$$p_f = \frac{n_f}{m}$$

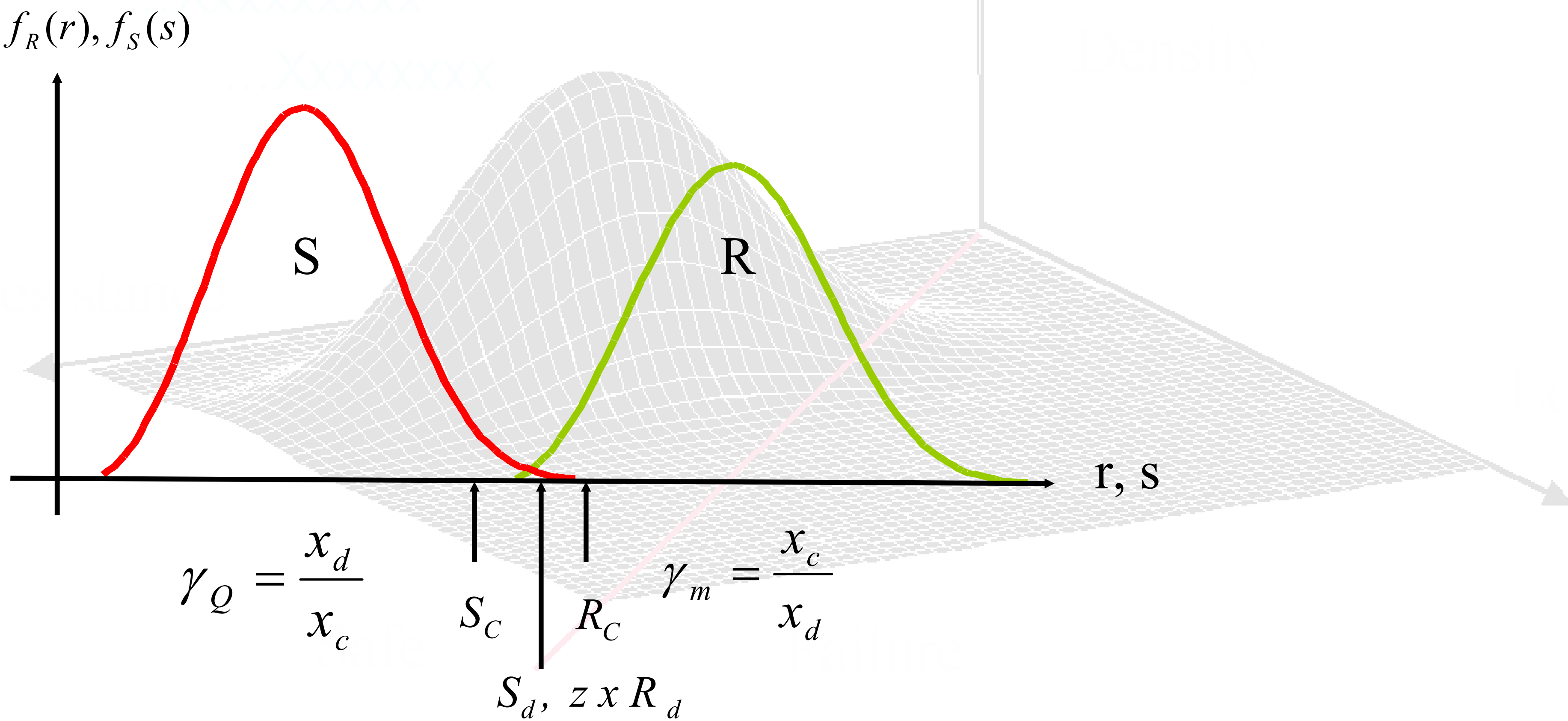


Structural reliability and safety formats

- The Load and Resistance Factor Design safety format is built up by the following components

Design situations	Ultimate, serviceability, accidental
Design equations	$g = \mathbf{z}R_c / \gamma_m - \left(\gamma_{G_a} G_c + \gamma_Q Q_C\right) = 0$
Design variables	$\mathbf{z}$
Characteristic values	$G_C \quad Q_C$
Partial safety factors	$\gamma_m \quad \gamma_G \quad \gamma_Q$
Design values	$\gamma_m = \dfrac{x_c}{x_d} \qquad \gamma_Q = \dfrac{x_d}{x_c}$

Basics of Structural Reliability Methods



Code calibration as a decision problem

- The code calibration problem can be seen as a decision problem with the objective to maximize the life-cycle benefit obtained from the structures by „calibrating“ (adjusting) the partial safety factors

$$\max_{\gamma} \quad W(\gamma) = \sum_{j=1}^{L} w_j \left[B_j - C_{Ij}(\gamma) - C_{Rj}(\gamma) - C_{Fj} P_{Fj}(\gamma) \right]$$

$$s.t. \quad \gamma_i^l \le \gamma_i \le \gamma_i^u \quad , i = 1,\dots,m$$

- The „optimal“ design is determined from the design equations

$$\min_{\gamma} \quad C_{Ij}(\mathbf{z})$$

$$s.t. \quad G_{\mathrm{j}}\left(\mathbf{x}_c, \mathbf{p}_j, \mathbf{z}, \gamma\right) \ge 0$$

$$\mathrm{z}_i^l \le z_i \le z_i^u \quad , i = 1,\dots,N$$

$$G_j(\mathbf{x}_c, \mathbf{p}_j, \mathbf{z}, \gamma) \ge 0$$

Target reliabilities for the design of structures

- Target reliabilities for Ultimate Limit State verification

Relative cost of safety measure	Minor consequences of failure	Moderate consequences of failure	Large consequences of failure
High	β=3.1 ($P_F \approx 10^{-3}$)	β=3.3 ($P_F \approx 5\ 10^{-4}$)	β=3.7 ($P_F \approx 10^{-4}$)
Normal	β=3.7 ($P_F \approx 10^{-4}$)	β=4.2 ($P_F \approx 10^{-5}$)	β=4.4 ($P_F \approx 5\ 10^{-5}$)
Low	β=4.2 ($P_F \approx 10^{-5}$)	β=4.4 ($P_F \approx 10^{-5}$)	β=4.7 ($P_F \approx 10^{-6}$)

- **Target reliabilities for Serviceability Limit State Verification**

Relative cost of safety measure	Target index (irreversible SLS)
High	β=1.3 ($P_F \approx 10^{-1}$)
Normal	β=1.7 ($P_F \approx 5\ 10^{-2}$)
Low	β=2.3 ($P_F \approx 10^{-2}$)

The JCSS approach to code calibration

- A seven step approach

1. Definition of the scope of the code
   - Class of structures and type of failure modes

2. Definition of the code objective
   - Achieve target reliability/probability

3. Definition of code format
   - how many partial safety factors and load combination factors to be used
   - should load partial safety factors be material independent
   - should material partial safety factors be load type independent
   - how to use the partial safety factors in the design equations
   - rules for load combinations

The JCSS approach to code calibration

- A seven step approach

4. Identification of typical failure modes and of stochastic model
   - relevant failure modes are identified and formulated as limit state functions/design equations
   - appropriate probabilistic models are formulated for uncertain variables

5. Definition of a measure of closeness
   - the objective function for the calibration procedure is formulated e.g.

$$\min_{\gamma} W(\gamma) = \sum_{j=1}^{L} w_j \left(\beta_j(\gamma) - \beta_t\right)^2 \qquad \min_{\gamma} W'(\gamma) = \sum_{j=1}^{L} w_j \left(P_{Fj}(\gamma) - P_F^t\right)^2$$

The JCSS approach to code calibration

- A seven step approach

6. Determination of the optimal partial safety factors for the chosen code format

7. Verification
   - incorporating experience of previous codes and practical aspects

The JCSS software CODECAL provides code calibration according to this approach

available on: www.jcss.ethz.ch

The JCSS Framework for Risk Assessment

- **Decisions and decision maker**

  A decision is:

  a committed allocation of resources

  the decision maker thus has responsibility for the committed resources – but also responsibility to any third party which may be affected by the decision

  the benefit of the decision should at least be in balance with the committed resources – this depends on the preferences of the decision maker – measured in terms of attributes

The JCSS Framework for Risk Assessment

- **Constraints on decision making**

  In principle – any society may define what they consider to be acceptable decisions

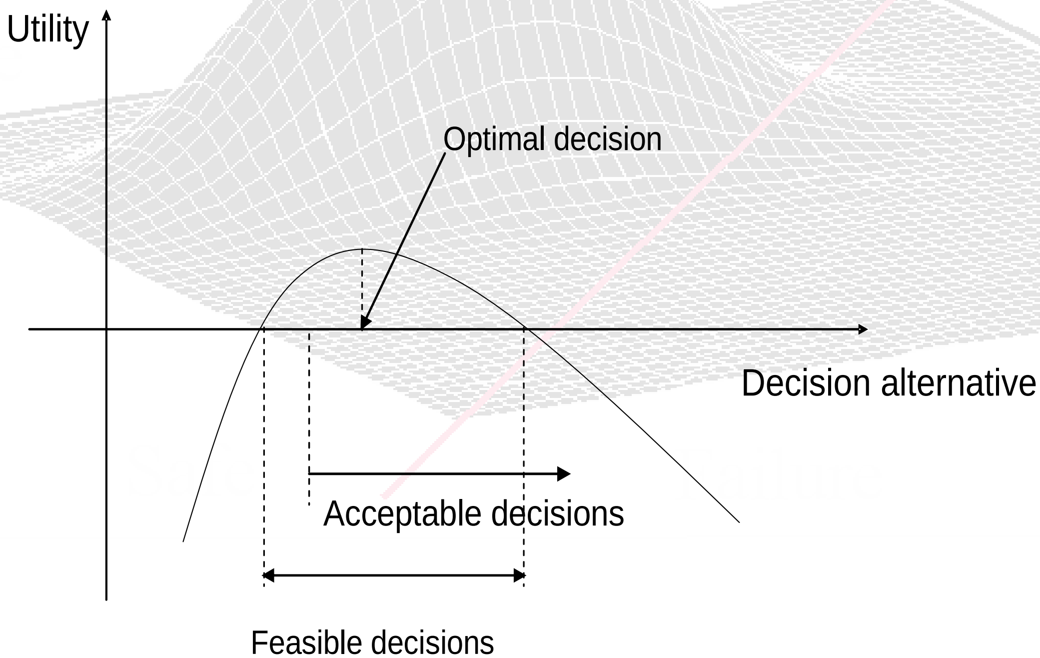
  Typically decisions are constrained – e.g. in terms of maximum acceptable risks to

  - persons
  - qualities of the environment

The JCSS Framework for Risk Assessment

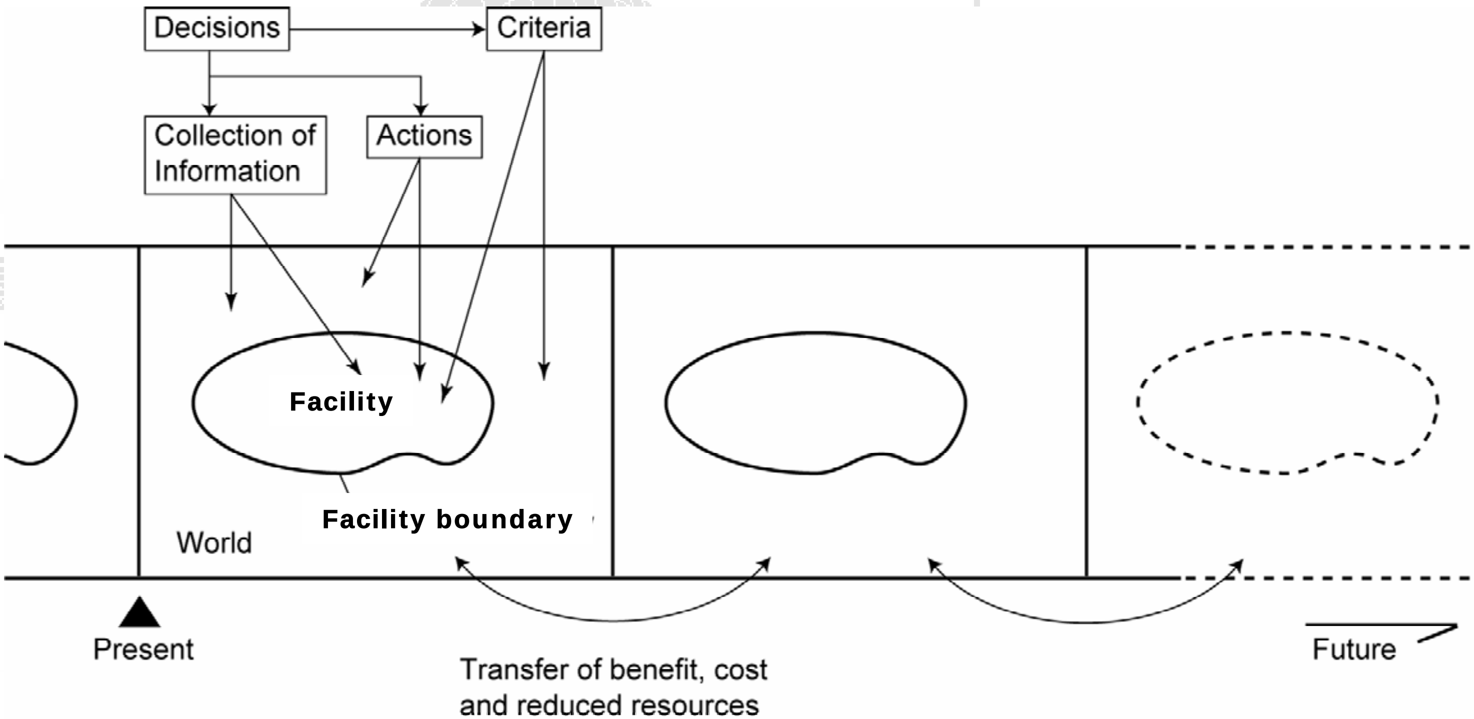
- **Feasibility and optimality**

  Feasible, optimal and acceptable decisions may be identified from



The JCSS Framework for Risk Assessment

- **System modelling**



The JCSS Framework for Risk Assessment

- **Knowledge and uncertainty**

  Remember that all uncertainties must be considered when the expected value of the utility is assessed

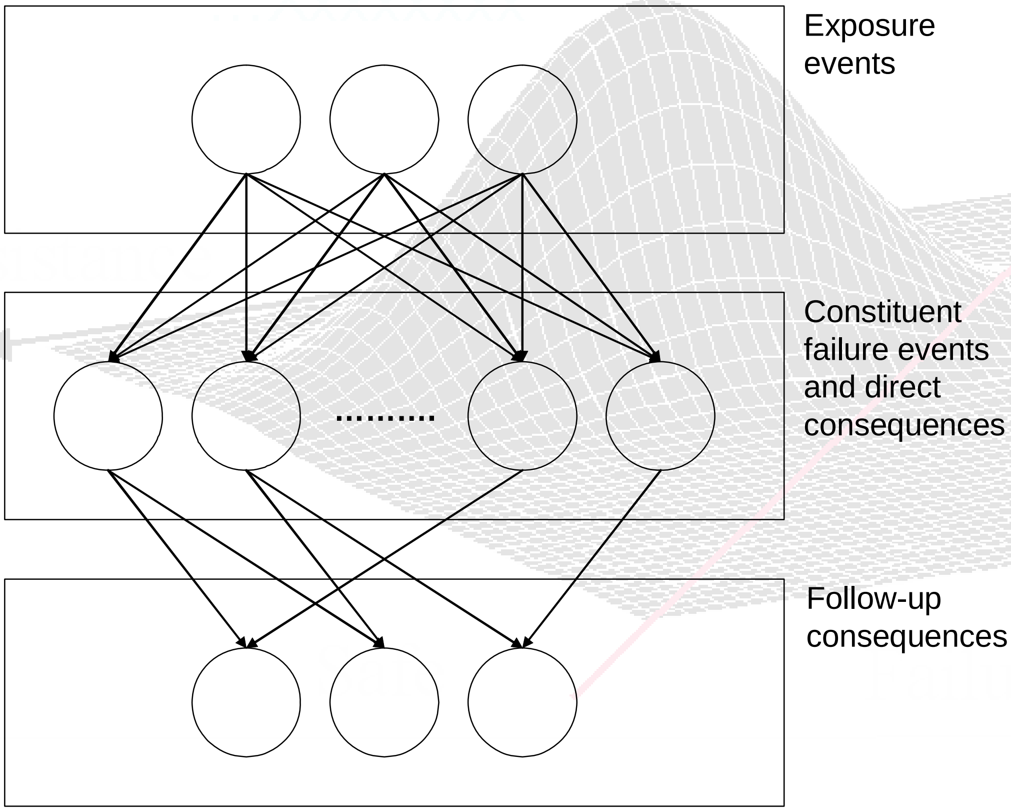
  - aleatory

  - epistemic

  It is important to address the possibility of the existence different system hypothesis – and take this into account in the decision problem

The JCSS Framework for Risk Assessment

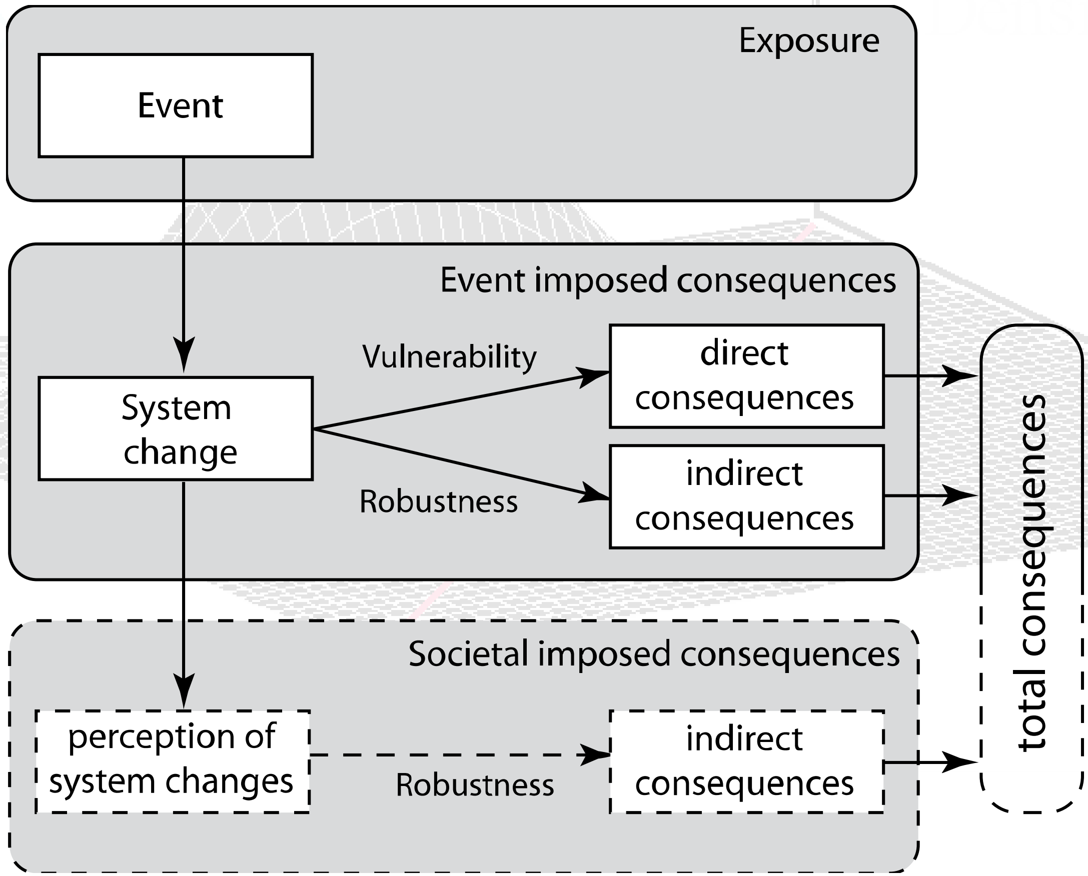
- System representation – scenarios of events



System representation must be refined enough to enable a comparison of the risks or benefits of different decision alternatives

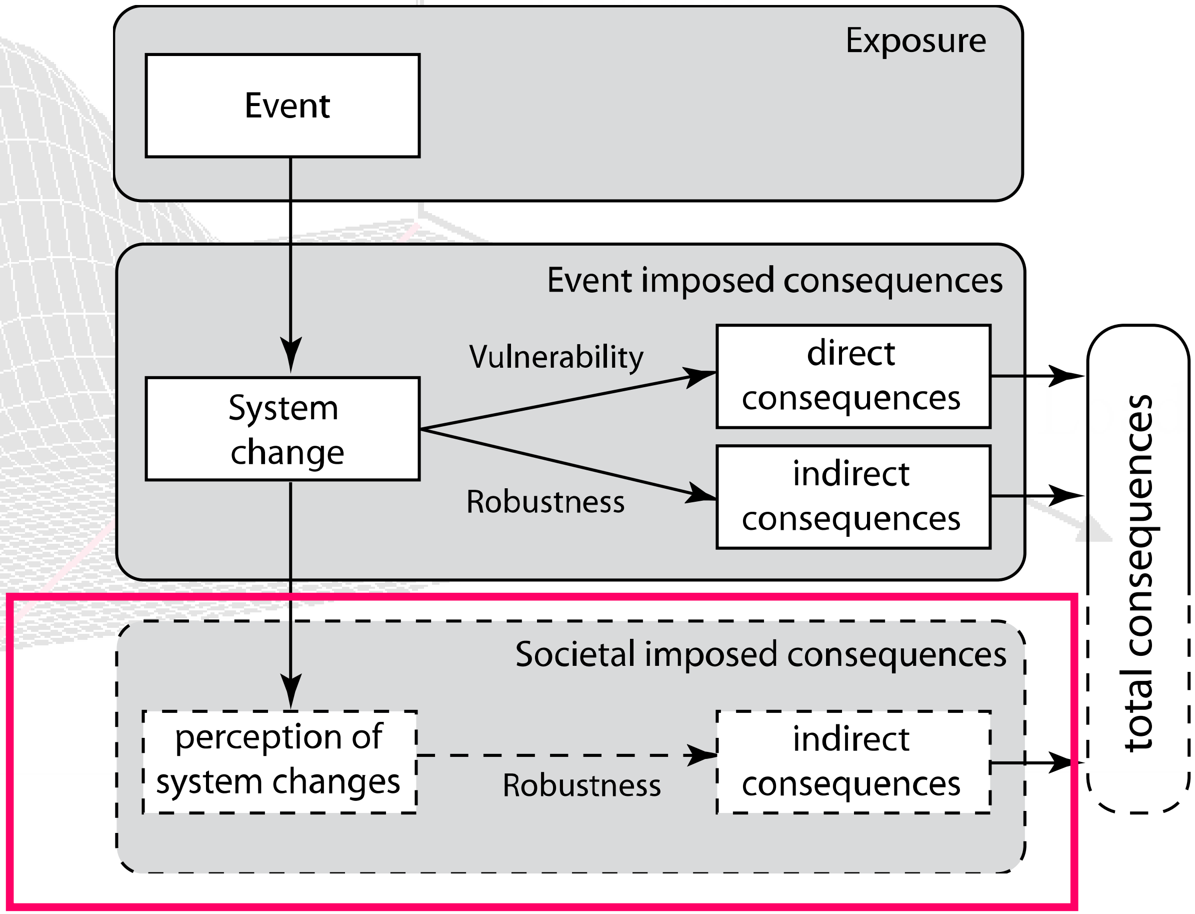
The JCSS Framework for Risk Assessment

- **System representation – evolution of consequences**



The JCSS Framework for Risk Assessment

- **Risk perception**



Due to perception of possible events

The JCSS Framework for Risk Assessment

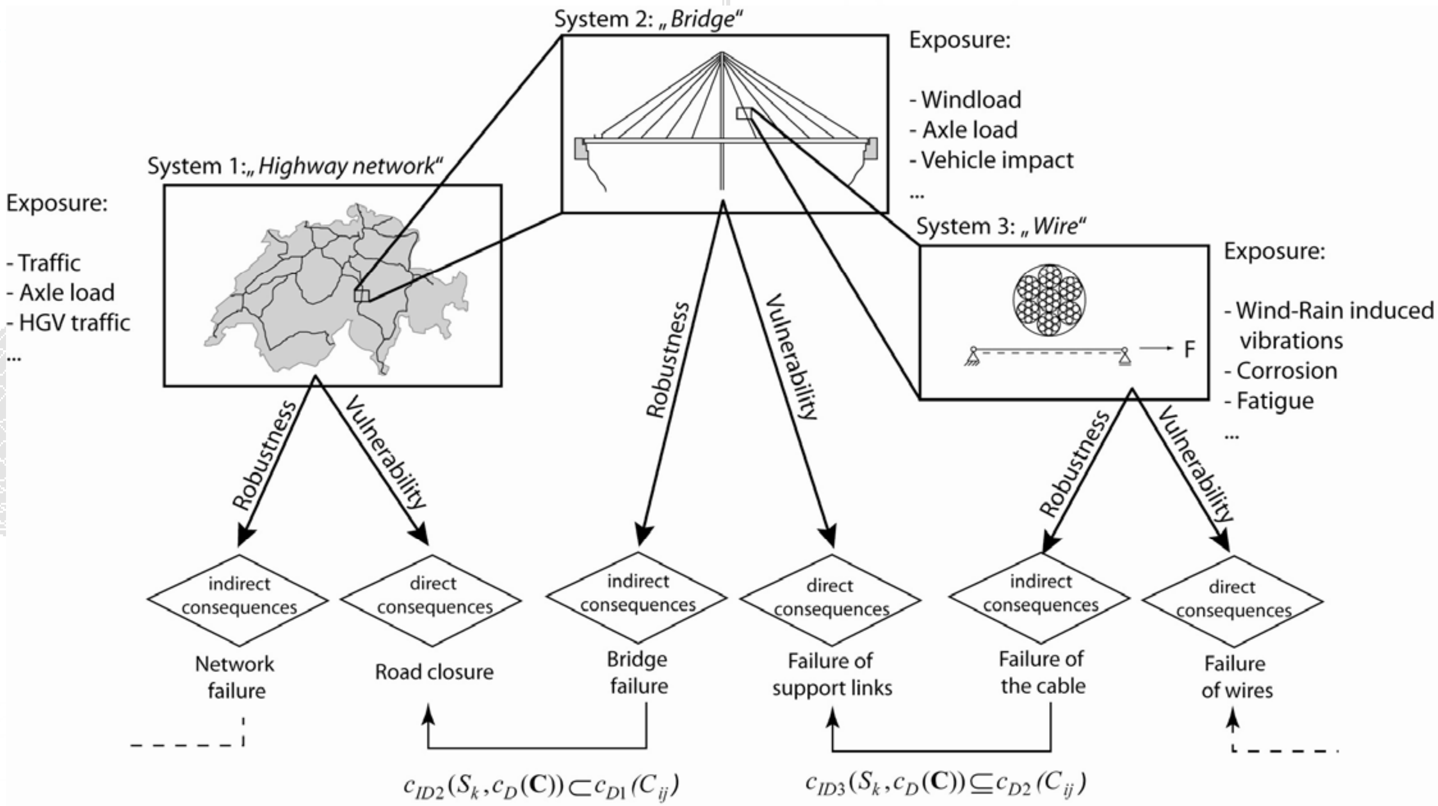
- **Comparison of decision alternatives**

Optimal decision alternatives are selected by comparing expected total utility

$$E\left[U(a_j)\right] =$$
$$\sum_{k=1}^{n_{EXP}} p(C_{ij} \left| EX_k, a_j) c_D(C_{ij}, a_j) p(EX_k, a_j) + \sum_{k=1}^{n_{EXP}} \sum_{l=1}^{n_{STA}} p(S_l \right| EX_k, a_j) c_{ID}(S_l, c_D(\mathbf{C}), a_j) p(EX_k, a_j)$$

The JCSS Framework for Risk Assessment

- **System representation – multiple scales**



The JCSS Framework for Risk Assessment

- **Assessment of risks**

Exposure	$p(EX_k)$
⇩	
Vulnerability	$p(C_{ij} \mid EX_k) c_D(C_{ij})$
⇩	
Robustness	$p(S_l \mid EX_k) c_{ID}(S_l, c_D(\mathbf{C}))$

Direct risks:

$$R_D = \sum_{k=1}^{n_{EXP}} p(C_{ij} \mid EX_k) c_D(C_{ij}) p(EX_k)$$

Indirect risks:

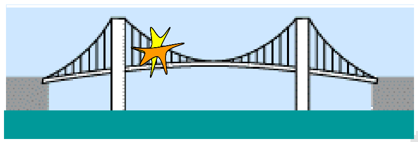
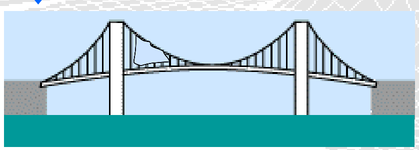
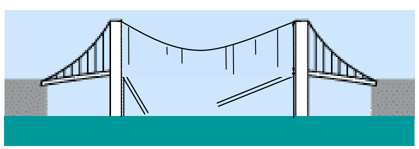
$$R_{ID} = \sum_{k=1}^{n_{EXP}} \sum_{l=1}^{n_{STA}} p(S_l \mid EX_k) c_{ID}(S_l, c_D(\mathbf{C})) p(EX_k)$$

Robustness Index:

$$I_R = \frac{R_D}{R_D + R_{ID}}$$

The JCSS Framework for Risk Assessment

- **Indicators of risks**

Scenario representation	Physical characteristics	Indicators	Potential consequences
Exposure 	Flood Ship impact Explosion/Fire Earthquake Vehicle impact Wind loads Traffic loads Deicing salt Water Carbon dioxide	Use/functionality Location Environment Design life Societal importance	
Vulnerability 	Yielding Rupture Cracking Fatigue Wear Spalling Erosion Corrosion	Design codes Design target reliability Age Materials Quality of workmanship Condition Protective measures	**Direct consequences** Repair costs Temporary loss or reduced functionality Small number of injuries/fatalities Minor socio-economic losses Minor damages to environment
Robustness 	Loss of functionality partial collapse full collapse	Ductility Joint characteristics Redundancy Segmentation Condition control/monitoring Emergency preparedness	**Indirect consequences** Repair costs Temporary loss or reduced functionality Mid to large number of injuries/fatalities Moderate to major socio-economic losses Moderate to major damages to environment

The JCSS Framework for Risk Assessment

- **Discounting**

  In evaluating the benefit and risk – the time of consequences as well as investments must be taken into account – by discounting

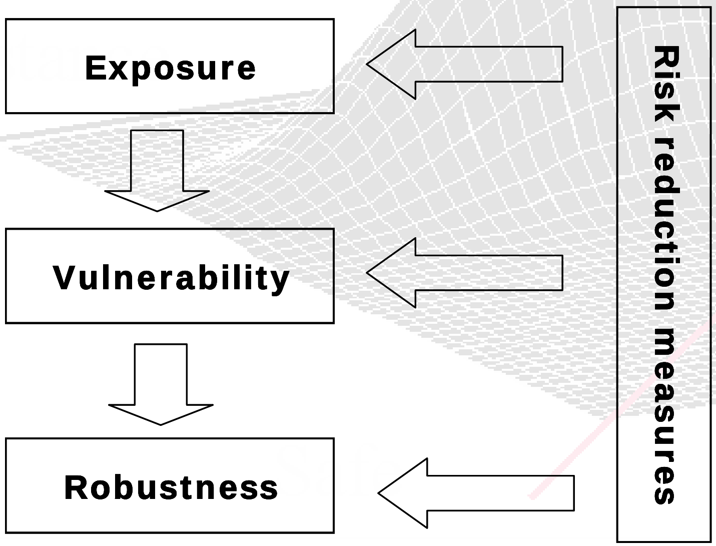
  - private discounting should consider long term investment return

  - public sector should consider only long term rate of economical growth – presently around 2 percent per annum

The JCSS Framework for Risk Assessment

- **Risk treatment – communication and transfer**

  - In principle risk may be treated at any level in the systems representation



Collecting more information

Changing the physical characteristics

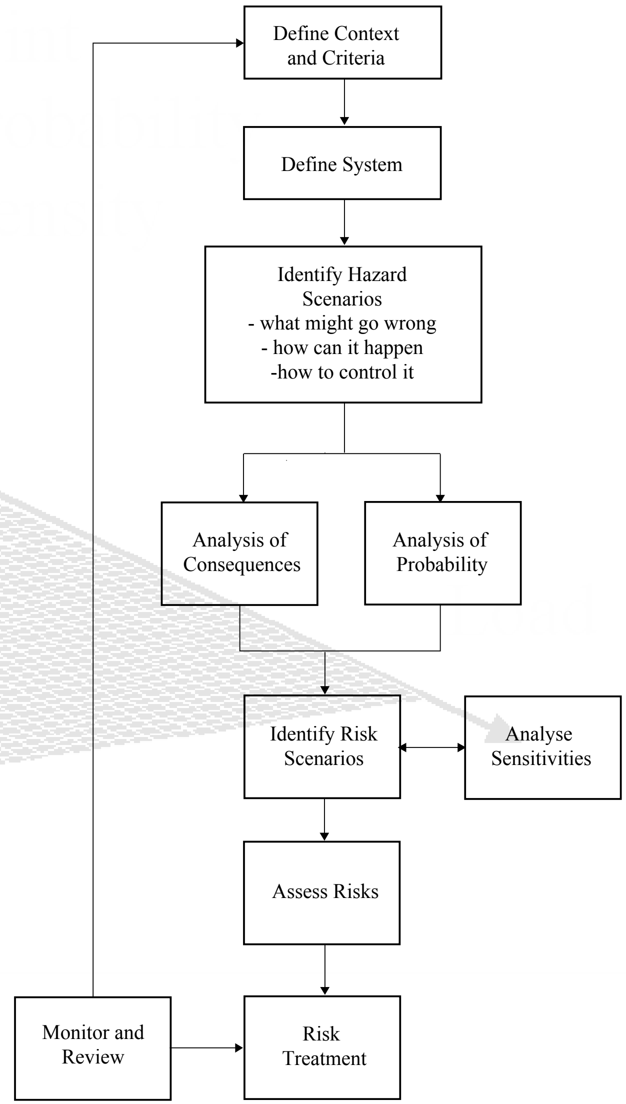
Risk information/communication

Transfer of risks - insurance

The Procedure of Risk Assessment

- **Basically the same steps should be performed for any type of facility/application area**

  Risk assessment procedures are generic



Life Safety – and the Performance of Society

- **Life safety is provided by many different sectors and through very different activities**

Risk reduction cost in SFr per saved person life	
100	Multiple vaccination - third world
$1 \cdot 10^3$	
$2 \cdot 10^3$	Medical X-ray facility
$5 \cdot 10^3$	Wearing motorbike helmet
$10 \cdot 10^3$	Cardiac ambulance
$20 \cdot 10^3$	Emergency helicopter service
$100 \cdot 10^3$	Safety belts in cars
	Crossway restructuring
to	
	Kidney dialysis
$500 \cdot 10^3$	Structural reliability
$1 \cdot 10^6$	
$2 \cdot 10^6$	
$5 \cdot 10^6$	City railway Zurich, Alp Transit
$10 \cdot 10^6$	Earthquake standard SIA
$20 \cdot 10^6$	Mine safety USA
$50 \cdot 10^6$	DC 10 out of service
$100 \cdot 10^6$	Multi-storey buildings regulation
$1 \cdot 10^9$	Removal of asbestos from public buildings

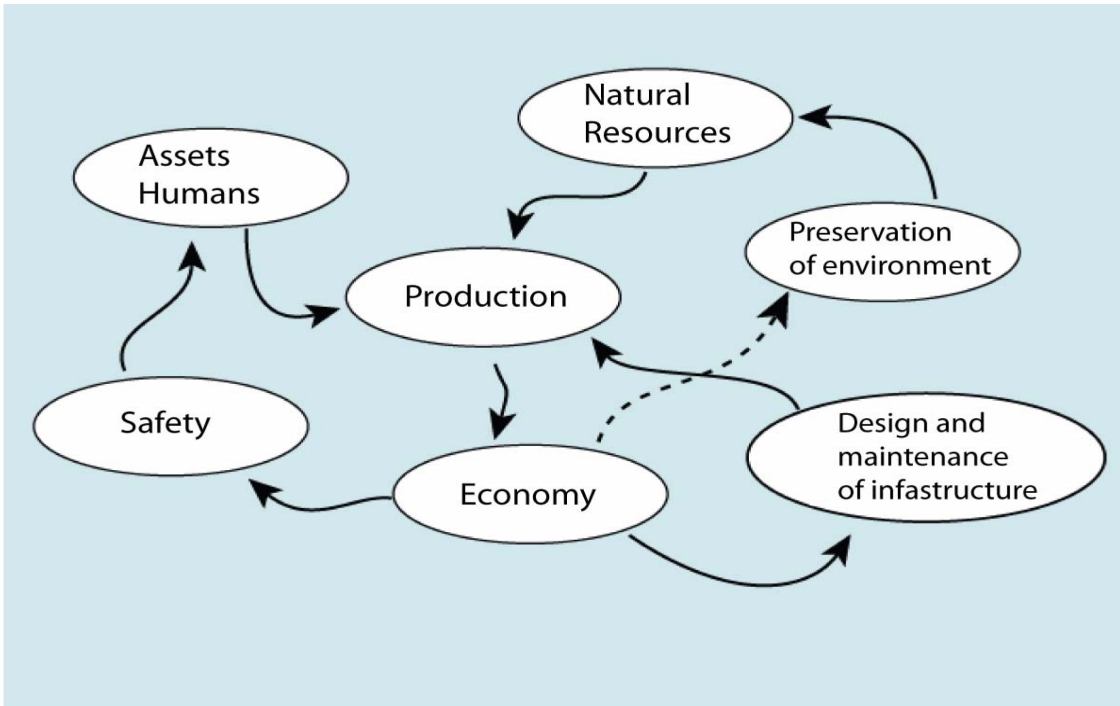
Efficiency is markedly different from sector to sector and from activity to activity !

It is a societal responsibility to spend public resources efficiently !

If this is not done – life is taken away from some individuals in society

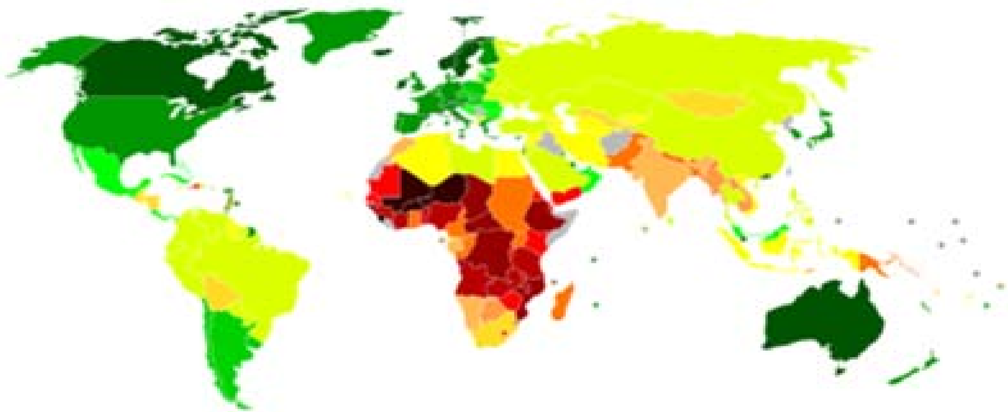
Life Safety – and the Performance of Society

- **Prioritization in society must be subject to a holistic perspective**



Life Safety – and the Performance of Society

- The performance of the nations of the world is measured through the *Human Development Index (HDI)*



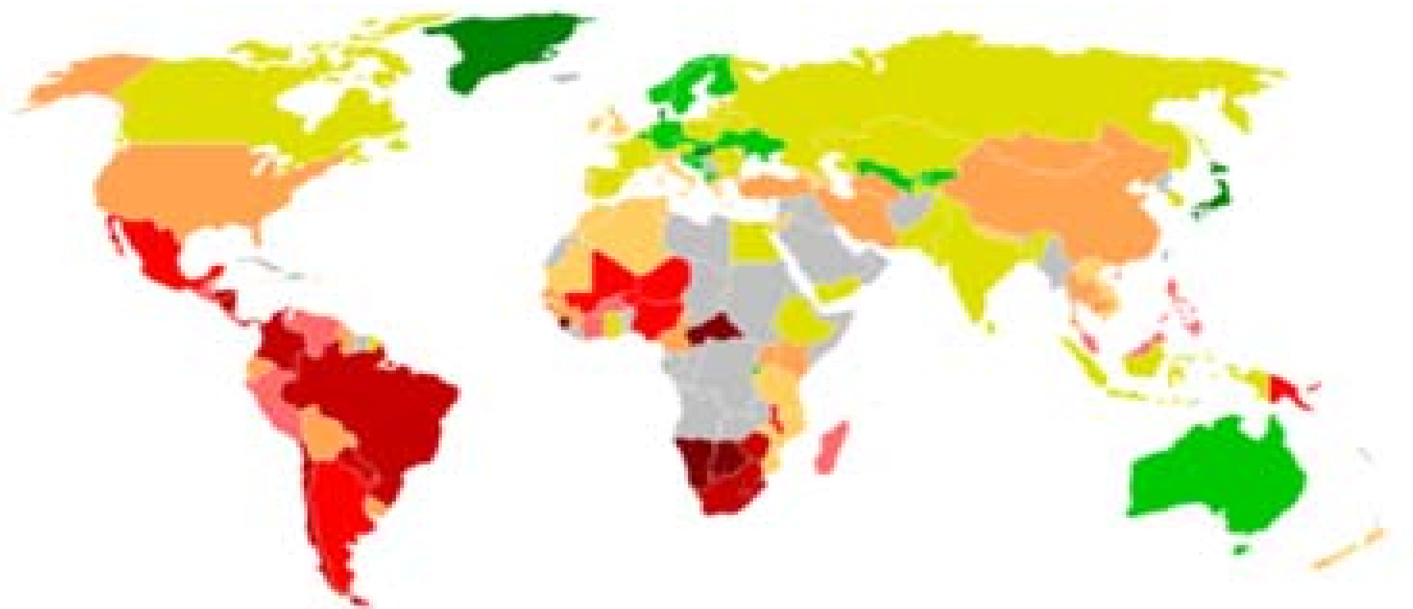
World map indicating Human Development Index (2004).

0.950 and over	0.700-0.749	0.450-0.499
0.900-0.949	0.650-0.699	0.400-0.449
0.850-0.899	0.600-0.649	0.350-0.399
0.800-0.849	0.550-0.599	0.300-0.349
0.750-0.799	0.500-0.549	under 0.300
		n/a

$$HDI = \frac{1}{3} GDP\ Index + \frac{1}{3} EI + \frac{1}{3} LEI$$

Life Safety – and the Performance of Society

- **It is also interesting to observe how the income of nations is distributed between the individuals of the nations (*Gini – Index*)**



Color	Gini coefficient
dark green	< 0,25
green	0,25 - 0,29
yellow-green	0,30 - 0,34
light orange	0,35 - 0,39
orange	0,40 - 0,44
pink-red	0,45 - 0,49
red	0,50 - 0,54
dark red	0,55 - 0,59
maroon	> 0,60
grey	NA

$$HDI = \frac{1}{3} GDP\ Index + \frac{1}{3} EI + \frac{1}{3} LEI$$

Modelling Socio-Economical Acceptable Risks

- **Taking basis in the philosophical insight that the basic asset individuals have is time – Nathwani, Pandey and Lind developed the *Life Quality Index* – a preference model – which at a societal level acts as a revealed preference on how we weight money against life time and time for private activities**

$$L(g,\ell) = g^q \ell$$

g : is the part of the GDP available for investment into life safety

ℓ : is the life expectancy at birth

w : is the part of life spent for work

$$q = \frac{1}{\beta}\frac{w}{1-w}$$

β : is a factor which takes into account that only a part of the GDP is based on humal labour

Modelling Socio-Economical Acceptable Risks

- **Based on the LQI – the consideration that every investment into life safety should lead to an increase in life-expectancy results in a risk acceptance criterion:**

$$\frac{dg}{g} + \frac{1}{q}\frac{d\ell}{\ell} \geq 0$$

which leads to the important Societal Willingness To Pay (SWTP) criterion:

$$SWTP = dg = -\frac{g}{q}\frac{d\ell}{\ell}$$

GDP	59451 SFr
l	80.4 years
w	0.112
β	0.722
g	35931 SFr
q	0.175

Modelling Socio-Economical Acceptable Risks

- **The SWTP criterion is readily applied for the purpose to determining acceptable structural failure probabilities**

$$\frac{d\ell}{\ell} \approx C_x d\mu = C_x k dm$$

where

C_x is a demographical constant

k is the probability of dying in case of structural failure

m is the failure rate of a considered structural system

Modelling Socio-Economical Acceptable Risks

- **The SWTP criterion is readily applied for the purpose to determining acceptable structural failure probabilities**

$$dC_y(p) \geq -\frac{g}{q} C_x N_{PE} k dm(p)$$

where

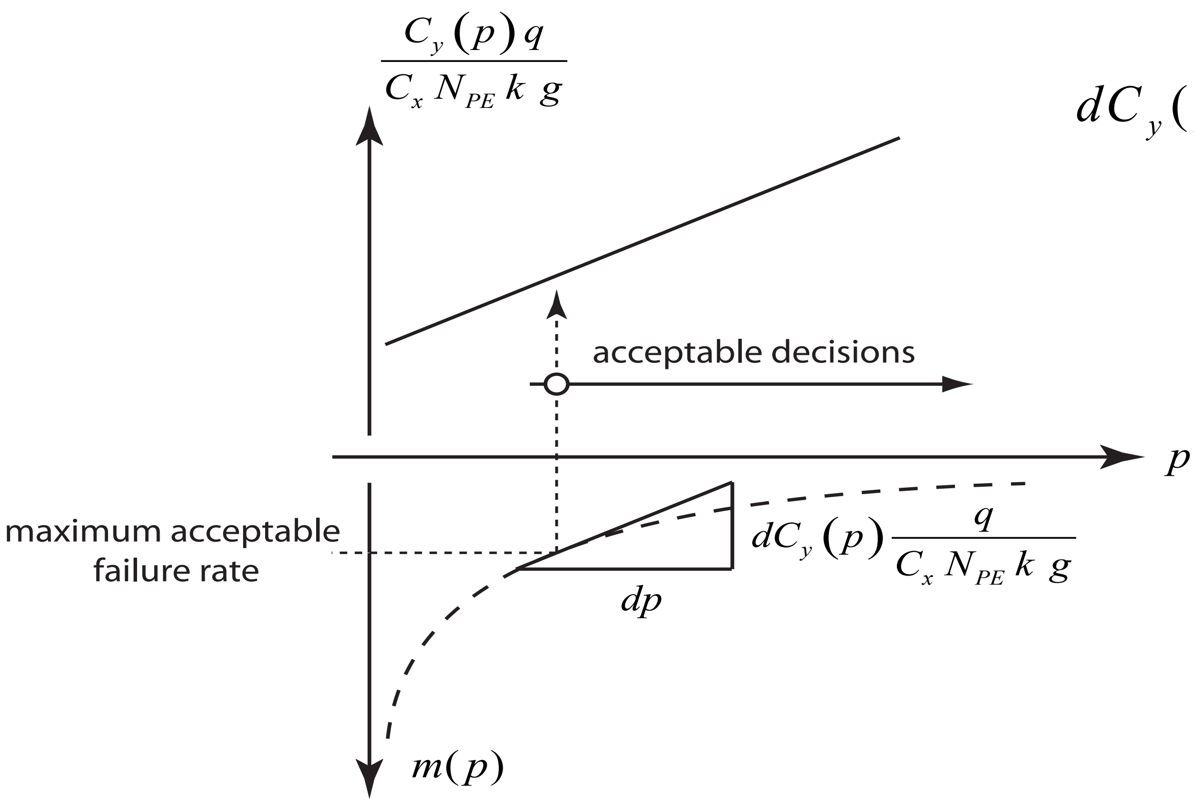
$dC_y(p)$ are the annual costs spent for risk reduction

N_{PE} is the number of people exposed to the structural failure

p is a decision alternative e.g. a structural dimension

Modelling Socio-Economical Acceptable Risks

- **The SWTP criterion can be visualized**



$$dC_y(p) \geq -\frac{g}{q} C_x N_{PE} k dm(p)$$

Modelling Socio-Economical Acceptable Risks

- **Based on the LQI – also the costs of compensation for a lost life can be assessed – Societal Value of a Statistical Life (SVSL).**

$$SVSL = \frac{g}{q} E$$

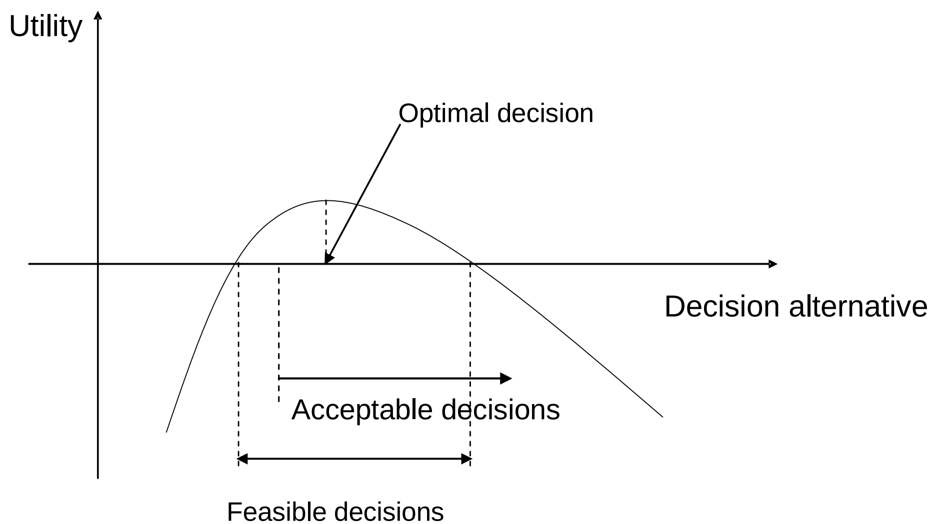
For Switzerland this amounts to about 6 million SFr

Modelling Socio-Economical Acceptable Risks

- **Now the optimization problem can be reassessed –**

  Acceptable decisions are limited by the SWTP criterion

  Costs of failure include compensation – through the SVSL



Reliability Assessment of Structures

Thanks for your attention !